Iranian Journal of Operations Research
Vol. 11, No. 1, 2020, pp. 107-121
DOI: 10.29252/iors.11.1.107

A Robust Optimization Model for the Hub Location and Revenue Management Problem Considering Uncertainties

Yaser Rouzpeykar [1], Roya Soltani [2,*], Mohammad Ali Afashr Kazemi [3]

The hub location and revenue management problem are two research topics in the field of network design and transportation. The hub location model designs the structure of the transportation network, while the revenue management model allocates network capacity to different customer categories according to their price sensitivity. Revenue management determines which products to sell to which customers and at what price. On the other hand, due to the limited number of aircraft seats, the revenue management problem has been widely used in the aviation industry. In this study, a robust optimization model is developed for the hub location and revenue management problem. For this purpose, a real-world case study with a central hub and six airports is presented and solved using CPLEX solver in GAMS software. Finally, a sensitivity analysis was performed on the key parameters of the problem, and their effect on the objective functions of the problem was investigated. Results show that the proposed model achieved the feasible solution in reasonable time for real case problem by exact method.

Keywords*: Hub location, revenue management, robust optimization, aviation industry*.

Manuscript was received on 02/10/2020, revised on 04/24/2020 and accepted for publication on 06/12/2020.

1. Introduction

Revenue management determines which products to sell to which customers and at what price (Sierag al., [1]). On the other hand, due to the limited number of aircraft seats, the revenue management problem has been widely used in the aviation industry. In the aerospace industry, most seats are offered at different prices to different customer categories (Çetiner, [2]).

Airlines categorize their customers and allocate different capacities to each category according to their prices to obtain maximum revenue. Capacity control in the aviation industry includes various models, algorithms, and policies for allocating seats to maximize the expected profit (Tikani et al., [3]).

The hub location problem is related to the placement of hub facilities and the allocation of demand nodes to determine the traffic route between the source and destination pairs. Nowadays, the hub location problem has been considered by many researchers because it has a significant effect on reducing the number of network connections and system costs. In the *p*-star hub-star network, the point *p* is selected as the hub. Each node is connected to precisely one hub, and all the hubs are connected to a central hub. The central hub is initially defined, but other hubs are determined by the model (Yaman, [4]).

[*] Corresponding Author.
[1] Department of industrial engineering, Qeshm branch, Islamic Azad University, Qeshm, Iran.
[2] Department of Industrial Engineering, KHATAM University,Tehran, Iran, r.soltani@khatam.ac.ir, orcid.org/0000-0002-1473-5337.
[3] Department of Industrial and information technology management, Tehran central Branch, Islamic Azad University, Tehran, Iran.

There are four types of hub location problems in the literature: the p-median hub location problem, the *p*-center hub location problem, the hub covering location, and the hub fixed cost location problem. In *p* center hub location problem, the goal is to locate the *p* hubs in such a way as to minimize the total cost of transmitting current across the network. The number of hubs in this problem is known. The *p*-center hub location problem searches for the optimal location of the *p* hubs and the allocation of non-hub points to hub points in conditions that the maximum path in the network is minimized. For the hub covering location problem, the number of hubs is not specified, and the demand points are covered if they are within a certain distance from the hub. This problem seeks to minimize the setup cost of the facility in conditions that all points are covered. In the hub location problem with a fixed cost, the setup cost of hubs is considered, and the number of hubs is unknown. Also, setting up a flow in the network is such that the flow transmission and the setup costs of hubs are minimized (Mohammadi and Tavakoli Moghaddam, [5]).

Several real-world optimization problems and engineering problems, in particular, are multi-objective optimization problems. The multi-objective nature of engineering problems has made them more challenging to solve and select an appropriate algorithm. Achieving an optimal balance between different goals is one of the main issues in the solution process. Most of these problems are combined with several different non-weighted and often non-directional objective functions. This means that the process of evaluating other functions cannot be essentially identical and in the same direction. In many cases, increasing the value of one objective function is only possible by decreasing the value of the other objective function. Therefore, there is a need for some kind of compromise between different objectives for the final decision. How these compromise works are crucial in making decisions.

Various approaches such as probabilistic, fuzzy and robust models have been proposed to deal with the uncertainties in the problems. In this study, two approaches of fuzzy set theory and robust optimization have been used for modeling and problem solving.

Fuzzy set theory is used more than other approaches due to the advantages expressed in various studies. Since the fuzzy approach does not require accurate and sufficient information, it provides a more efficient model than other methods, such as the probabilistic approach, which requires sufficient knowledge of the distribution of uncertain parameters. This means that it is necessary to determine the distribution of the problem parameters in probabilistic methods and then determine their values, which is very difficult than the fuzzy approach (Balin, [6]). In conditions that the parameters of the problem are uncertain, a fuzzy scheduling algorithm can create an actual flexible system (Behnamian and Ghomi, [7]). Also, the computational complexity of fuzzy modeling is much less than other approaches (Slovensky and Hubb, [8]).

On the other hand, robust optimization planning offers a risk-averse approach to dealing with uncertainties in optimization problems. According to Pishvaee et al. [9], a solution to a robust optimization problem can be achieved only when it is feasibility and optimality robust at the same time. Feasibility robust means that the proposed solution must remain feasible for (almost) all possible values of the uncertain parameters and robust optimality means that the value of the objective function for (almost) all possible values of the uncertain parameters is close to the optimal value or at least has an insignificant deviation from the optimal value.

According to the description provided in this study, an integrated hub location and revenue management model will be presented for the aviation industry by considering uncertainties. The objective function of this study includes maximizing revenue from the transportation network and minimizing the setup cost of the hub. In this regard, from *n* potential hubs, *p* hubs are selected to connect the hubs by a central node. Communication capacity between hub and central hub nodes and hub and non-hub nodes is limited. The novelty of the present study is as follows:

- A robust optimization model for the integrated hub location and revenue management in the aviation industry
- Considering the integrated problem in aviation industry under uncertainty

- Presenting a real case study to evaluate the proposed model

2. Literature review

Hub location problems are divided into different types. The most important categorization that many authors have mentioned in their works divides hub location into three categories: center hub location problem, median hub location problem, and hub covering location. The objective of the center hub location problem is to minimize the maximum distance between the hub centers and the demand points (minimum-maximum objective function). The middle hub objective is to minimize the sum of the distances of the demand points to the hub centers (transportation system costs). However, in the hub covering location problem, the issue of limitations on the connection of normal nodes to hubs is raised according to the distances or transportation cost between them. In the hub location problem, it is assumed that hub facilities are always available, and no failure occurs. But in reality, some hubs may not be available due to failure. When a hub fails, the non-hub points assigned to that point must be allocated to other hub points, which results in additional cost. Therefore, some backup hub points to support the main hub are vital for the transportation of hazardous materials (Parvaresh et al., [9]; Mohammadi and Tavakoli Moghaddam, [5]).

Yaman and Elloumi [10] considered the star-star network for the central star hub and the middle star hub problems. They tried to minimize the distance of the longest path and the total routing costs in both problems. Xiao et al. [11] studied the airport capacity selection problem by considering demand uncertainty and the objective of maximizing profits and welfare in the aviation industry. Azizi et al. [12] proposed a hub location model at the risk of a hub failure. They assumed that when a failure occurs in a hub, all the demand supplied by this hub is transferred to a backup hub. Hörhammer [13] studied allocating multivariate and multi-period single hub location problems. In this study, it is assumed that the hub can be closed or resized, and one non-hub node in each period can become a hub. For this purpose, four models of mixed integer quadratic mathematical programming based on flow and route were developed. The purpose of this study is to minimize the cost of communication between non-hub and hub points, the cost of transportation from one hub to other hubs, the cost of distribution from one hub to non-hub points, the cost of setting up and closing hub, and the cost of resizing hub.

Adibi and Razmi [14] presented a two-stage stochastic model for allocating multiple hubs problems without capacity constraints, considering the uncertainty of the demand and cost of transportation. In order to evaluate the proposed model, a case study including ten nodes of the best cities in Iran’s air transport network has been considered. Damgacioglu et al. [15] proposed a genetic algorithm to solve the hub location problem without capacity. Grauberger and Kimms [16] studied the airline revenue management problem by considering the behavior of competitors in terms of capacity and competitive price. He [17] investigated the effect of revenue management problem on the hub-to-hub network. Alumur et al. [18] studied multi-period hub location problem for single and multiple allocations. In each period, new hub size and capacity development of existing hubs are allowed. For this purpose, a mixed-integer linear programming model has been developed to minimize transportation costs, communication between hubs, capacity building, and hub setup. Tikani et al. [3] proposed an integrated model for the hub location and revenue management problem, taking into account several customer levels in the aviation industry. The purpose of this study was to maximize revenues and minimize costs. For this purpose, a two-step stochastic model for determining hub locations has been developed, and an efficient modified genetic algorithm has been implemented to solve the large-scale problem. The computational results showed the efficiency of the proposed algorithm. Alumur et al. [19] analyzed the hub location problem with single and multiple allocation capacities. In this study, a direct relationship between two non-hub points is considered. To solve the proposed problem, a mixed-integer linear programming model has been developed to minimize transportation and operation costs of hubs with different capacities. He et al. [17] presented an integrated model of p hub location and revenue management with multiple capacity levels and in the discontinuity conditions. For this purpose, a two-stage stochastic planning model has been developed to maximize network profit, in which the cost of setting up the hub at different levels, transportation costs, and revenues from ticket sales are considered. In order to achieve adequate solutions, a combined model of robust optimization and two-stage stochastic programming with an emphasis on maximizing total weighted profit has been developed. A genetic algorithm is also proposed to solve the hybrid model. Hou et al. [20] examined an integrated problem of hub location and revenue management and analyzed combined average and worst-case modes. In this regard, among the available nodes, the p hub is allocated. The used data in this paper was uncertain, and a limited set of scenarios was considered. For this purpose, a two-stage stochastic planning model was developed to maximize profits. The objective function was included the cost of setting up the hubs, the weighted sum of the transportation costs of the two average and worst-case modes, and the revenues from ticket sales in all scenarios. Salehi and Tikani [21] studied a capacitated reliable hub network design considering revenue management under traffic uncertainty. They developed a hybrid algorithm based on genetic operators to find near optimal solutions. Cvokic and Stanimirovic [22] studied a single allocation hub location and pricing problem to maximize the totoal profit. They developed a mixed-integer linear programming model for the deterministic situation. Then, a robust optimization model is proposed for the uncertain counterpart case. Rahmati and Neghabi [23] developed a balanced hub location problem uncer transportation cost uncertainty. They proposed a two-stage robust optimization model including mixed-integer linear and non-linear structures. The Benders decomposition algorithm is used to obtain Pareto solutions. Tiwari et al. [24] studied a hub location problems faced by an airline entering a competitive market. They developed a non-linear integer programming model and used Lagrangian relaxation method to obtain solutions. Willey and Salmon [25] developed a novel approach for urban air mobility network design using hub location. To do this end, five heuristic algorithms are developed to find potential solutions. Table 1 summarizes the most recent studies in the field of hub location and revenue management problem.

Table 1. A summary of reviewed papers

Authors' name	Publication year	Scope		Objective function		Model type		
		revenue management	location	revenue maximization	Cost minimization	robust	stochastic	deterministic
Yuan et al	2014		✓		✓	✓		
Hourhammer	2014		✓		✓			✓
Yuan et al	2015		✓		✓			✓
Adibi and Razmi	2015		✓		✓		✓	
Grauberger and Kims	2016	✓		✓				✓
Hee	2016	✓		✓				✓
Almore et al	2016		✓		✓			✓
Tikani et al	2018	✓	✓	✓	✓		✓	
Almore et al	2018		✓		✓			✓
Hu et al	2019	✓	✓	✓	✓		✓	
Hu et al	2019	✓	✓	✓	✓		✓	
Salehi and Tikani	2020		✓	✓	✓			✓
This paper		✓	✓	✓	✓	✓		

3. Proposed mathematical model

In this study, the integrated problem of hub location and revenue management in the aviation industry will be presented by considering uncertainty conditions. The structure of studied network is shown in Figure 1. There is a central hub and several nodes where some of them are selected as hub nodes. The objective of the problem is to maximize revenues from the transportation network and minimize total costs. In this section, the proposed model, including problem definition, assumptions, notation, and mathematical model, will be presented. In this study, there is a central hub that connects to several other hub points. There are many candidate points for hub nodes, from which the p points must be selected as the hub. Each of the non-hub points should then be connected to one of the hub points. The purpose of this problem is to minimize the total cost of transportation and operation of hubs so as to maximize revenue from ticket sales. Based on the airplane capacity, the route is determined from the hub point to the non-hub point so that the demand of each demand point is provided. The assumptions of the problem are as follows.

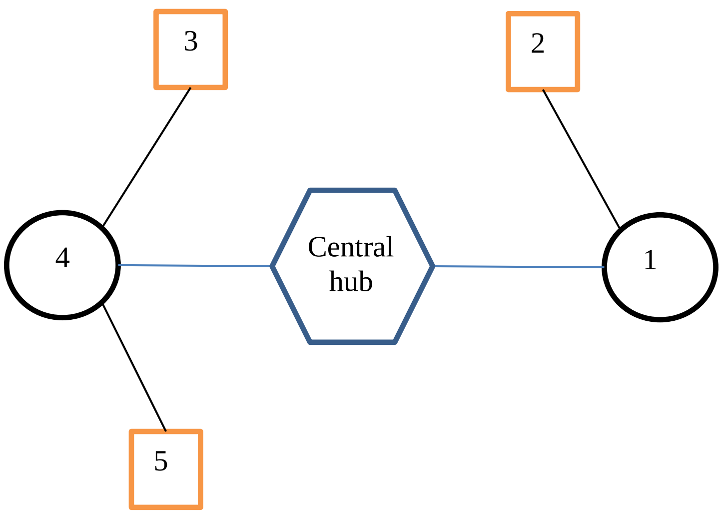

Figure 1. The proposed network design

3.1. Assumptions

The assumptions considered in this issue are as follows.

- All origin and destination points are candidates for selection as hub points.
- The number of hubs is determined from the beginning.
- The location of the central hub is known from the beginning.
- Each node is assigned to only one hub.
- Both nodes should be connected directly to each other but should be done by a hub.
- Transport between hubs is done by the central hub. In other words, there is no direct connection between the two hubs.
- The number of flights between two nodes is limited.
- Aircraft have different capacities.
- Additional luggage is allowed for passengers.
- It is possible to carry goods in addition to the passenger.

3.2. Sets and indices

N	Number of nodes
p	Number of hubs $p \subset N$
K	Number of flight classes
i, m	Index of node $i, m = 1, 2, ..., N$
j	Index of hub $j = 1, 2, ..., P$
k	Index of flight class $k = 1, 2, ..., K$

3.3. Parameters

dis_{j0}	Distance between the central hub and hub *j*
dis_{ij}	Distance between hub *j* and non-hub node *i*
c_{j0k}	Unit cost of travel between the central hub and hub *j*
c_{ijk}	Unit cost of travel between hub *j* and non-hub node *i*
cap_{j0}	Number of available flights from the central hub to hub *j*
cap_{ij}	Number of available flights from hub *j* to non-hub node *i*
p_{imk}	Ticket price from node *i* to node m with flight class *k*
ph_{imk}	Unit price of the additional load from node *i* to node m with flight class *k*
pg_{im}	Unit price of goods from node *i* to node *m*
d_{imk}	Demand from node *i* to node *m* with flight class *k*
vh_{imk}	The amount of additional load from node *i* to node *m* with flight class *k*
vg_{im}	Goods transported from node *i* to node *m* with flight class *k*
cl_1	Path capacity between central hub and other hubs
cl_2	Path capacity between hubs and non-hub points
fc_0	Fixed cost of setting up a central hub
fc_j	Fixed cost of setting up a hub *j*
A	A large integer
λ_{im}	The level of confidence of the decision-maker for the path between nodes *i* and *m*
a_{ij}	An independent stochastic variable
$\hat{a}_{ij}$	A deviation from the nominal coefficient a_{ij}
d_j	The deviation from the nominal coefficient
Γ_i	the protection level for the *i*-th block.
$\bar{d}_{imk}$	he estimated value of the customer demand parameter
$\hat{d}_{imk}$	The fluctuation rate of the demand parameter

3.4. Variables

x_{imk}	Number of tickets sold between nodes *i* and *m* with flight class *k*
y_{imk}	Level of protection between nodes *i* and *m* with flight class *k*

z_{ij}	If the node non-hub i is assigned to hub j 1 and otherwise 0
z_{jj}	If node i is selected as a hub 1 and otherwise 0
o_{im}	If there is a flight between nodes i and m, otherwise 0

3.5. Mathematical model

The proposed mathematical model is formulated as follows.

$$\max \quad z_1 = \sum_{i=1}^{N}\sum_{m=1}^{N}\sum_{k=1}^{K} p_{imk} \times x_{imk} + \sum_{i=1}^{N}\sum_{m=1}^{N}\sum_{k=1}^{K} ph_{imk} \times vh_{imk} \times o_{im} + \sum_{i=1}^{N}\sum_{m=1}^{N} pg_{im} \times vg_{im} \times o_{im} \quad (1)$$

$$\min \quad z_2 = \sum_{j=1}^{P}\sum_{k=1}^{K} c_{j0k}\left[dis_{j0}\left(\sum_{i=1}^{N}\sum_{m=1,m\neq i}^{N}\left(\frac{x_{imk}}{cl_2}\right) z_{ij}(1-z_{mj})\right) + dis_{0j}\left(\sum_{i=1}^{N}\sum_{m=1,m\neq i}^{N}\left(\frac{x_{imk}}{cl_2}\right) z_{ij}(1-z_{mj})\right)\right] + \sum_{i=1}^{N}\sum_{j=1}^{P}\sum_{k=1}^{K} c_{ijk}\left[dis_{ij}\left(\sum_{m=1,m\neq i}^{N}\frac{x_{imk}}{cl_1}\right) + dis_{ji}\left(\sum_{m=1,m\neq i}^{N}\frac{x_{imk}}{cl_1}\right)\right] z_{ij} + \sum_{j=1}^{P} fc_j z_{jj} + fc_0 \quad (2)$$

s.t.

$$\sum_{j=1}^{P} z_{ij} \le 1 \qquad \forall i = 1,2,\ldots,N \quad (3)$$

$$\sum_{j=1}^{N} z_{jj} = P \quad (4)$$

$$z_{ij} \le z_{jj} \qquad \forall i = 1,2,\ldots,N, \qquad \forall j = 1,2,\ldots,P \quad (5)$$

$$x_{imk} \le d_{imk} \qquad \forall i,m = 1,2,\ldots,N, \qquad \forall k = 1,2,\ldots,K \quad (6)$$

$$x_{imk} \le y_{imk} \qquad \forall i,m = 1,2,\ldots,N, \qquad \forall k = 1,2,\ldots,K \quad (7)$$

$$\sum_{k=1}^{K} x_{imk} \ge o_{im} \qquad \forall i,m = 1,2,\ldots,N \quad (8)$$

$$\sum_{k=1}^{K} x_{imk} \le A \times o_{im} \qquad \forall i,m = 1,2,\ldots,N \quad (9)$$

$$\sum_{i=1}^{N}\sum_{j=1}^{P}\sum_{k=1}^{K}\left(\frac{y_{imk}}{cl_2}\right) z_{ij}(1-z_{mj}) + \sum_{i=1}^{N}\sum_{j=1}^{P}\sum_{k=1}^{K}\left(\frac{y_{mik}}{cl_2}\right) z_{ij}(1-z_{mj}) \le cap_{j0} \times z_{jj} \qquad \forall j = 1,2,\ldots,P \quad (10)$$

$$\sum_{m=1}^{N}\sum_{k=1}^{K}\left(\frac{y_{imk}}{cl_1}\right)+\sum_{m=1}^{N}\sum_{k=1}^{K}\left(\frac{y_{mik}}{cl_1}\right)\le\sum_{j=1}^{P}cap_{ij}\times z_{ij}+A\times z_{ii}\qquad\forall i=1,2,...,N \tag{11}$$

$$z_{ij},o_{im}\in\{0,1\}\qquad\forall i,m=1,2,...,N,\qquad\forall j=1,2,...,P \tag{12}$$

$$x_{imk},y_{imk}\ge 0\qquad\forall i,m=1,2,...,N,\qquad\forall k=1,2,...,K \tag{13}$$

Equation (1) maximizes the total revenues obtained from sold tickets in different classes, carrying an extra cargo of passengers and carrying goods. Equation (2) minimizes the total cost of the network, including the cost of transportation between nodes and the cost of setting up hubs. The profits from the network are obtained from the difference between the above two expressions.

Equation (3) ensures that each non-hub node is assigned to only one hub. Equation (4) states that there are exactly *p* hubs in the network. Equation (5) indicates that a non-hub node is assigned to a node when that node is selected as the hub. Equations (6) and (7) represent that the maximum allowable amount of sold tickets is equal to the amount of demand and protection level, respectively. Equations (8) and (9) indicate that a flight between two nodes occurs when a ticket has been sold for that route. Equation (10) states that the protection level should not exceed the physical capacity between the central hub and other hubs. It means that the number of passenger carried by plane in both route from central hub to other hubs or from hubs to other nodes have not exceeded than the number of passengers considered as protection level. Equation (11) represents that the level of protection should not exceed the physical capacity between hubs and non-hub points. Finally, Equations (12) and (13) express the nature of the problem's decision variables.

4. Robust optimization

In this study, a robust optimization approach is used to solve the proposed problem. Robust optimization planning provides a risk-averse approach to deal with uncertainties in optimization problems. According to Pishvaee et al. [26], a solution to a robust optimization problem can be achieved only when it is feasibility and optimality robust at the same time. Feasibility robust means that the proposed solution must remain feasible for (almost) all possible values of the uncertain parameters, and robust optimality means that the value of the objective function for (almost) all possible values of the uncertain parameters is close to the optimal value or at least has an insignificant deviation from the optimal value. In this study, the proposed approach of Bertsimas et al. [27] is used because of developing a linear mathematical model and considering a controllable level of conservatism close to real-world conditions.

We will further explain the Bertsimas et al. [27] model for the linear optimization problem. In this model, the objective function is minimized, and the uncertainty coefficients have existed in both the objective function and the constraints in order to be more consistent with the original research model. We considered the following optimization problem in general:

$$\begin{aligned}&Min\ c^T x\\&Subject\ to\quad Ax\le b\\&l\le x\le u\end{aligned} \tag{14}$$

Uncertainty intervals are defined as follows:

Each of the constraint coefficients $a_{ij}, j\in N=\{1,2,\dots,n\}$ is modeled as an independent stochastic variable with a symmetric but unknown distribution $\hat{a}_{ij}, j\in N$ in the interval $[a_{ij}-\hat{a}_{ij}, a_{ij}+\hat{a}_{ij}]$ is a value that $\hat{a}_{ij}$ indicates a deviation from the nominal coefficient a_{ij}. Each of the coefficients of the objective function is set as $c_j, j\in N$ in the range $[c_j-d_j, c_j+d_j]$, where d_j represents the deviation from the nominal coefficient c_j. It should be noted that since the type of the objective function is minimization, and the goal of robust models is to obtain the maximum regret, only one side of the interval is used, i.e., it is assumed that c_j takes value in the interval $[c_j, c_j + d_j]$. To formulate a robust counterpart of the problem, Γ_i is defined as follows.

Consider the *i*-th constraint of the problem as $a_i^T x \leq b_i$, J_i is defined as a set of uncertain coefficients in line i. For each line *i*, we define the parameter Γ_i which is not necessarily an integer, such that $\Gamma_i \in [0, |J_i|]$. In fact, the role of Γ_i in constraints is to regulate the robustness of the proposed method. In fact, the proposed method is the conservatism level of the solution. However, it is unlikely that all coefficients could be uncertain together (Bertsimas et al. [27]). Therefore, we assume that the maximum value changes to $[\Gamma_i\text{-}\lfloor \Gamma_i \rfloor] a_{it}$. In other words, we assume that only a subset of the coefficients will be allowed to affect the obtained solution adversely. Considering this assumption, it is guaranteed that under the occurrence of such coincidence, in reality, the robust optimal solution will be definitely feasible. Also, due to the symmetric distribution of the variables, even if the number of varying coefficients exceeds $\lfloor \Gamma_i \rfloor$, the optimal answer will be feasible with high probability. Therefore, we call Γ_i the protection level for the *i*-th block.

The parameter Γ_0 controls the level of stability in the objective function. Therefore, we want to find the optimal solution in the cases where Γ_0 changes from the objective function coefficients and has the greatest effect on the solution.

Accordingly, the robust counterpart of the nominal linear optimization is obtained as follows (Bertsimas et al. [27]):

$$min \quad c^T x + \max_{\{s_0 | s_0 \subseteq J_0, |s_0| \leq \Gamma_0\}} \left\{ \sum_{j \in s_0} d_j |x_j| \right\} \tag{15}$$

S.t.

$$\sum_j a_{ij} x_j + \max_{\{s_i \cup \{t_i\} | s_i \subseteq J_i, |s_i| \leq \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus s_i\}} \left\{ \sum_{j \in s_i} \hat{a}_{ij} |x_j| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} |x_{t_i}| \right\} \leq b_i$$

$$l \leq x \leq u$$

If we want to transform the above model into a linear optimization model, the following theorem is needed.

Theorem: *for the vector x^*, the protection function of the i-th limit obtained from the following equation:*

$$\beta_i(x^*, \Gamma_i) = \max_{\{s_i \cup \{t_i\} | s_i \subseteq J_i, |s_i| \leq \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus s_i\}} \left\{ \sum_{j \in s_i} \hat{a}_{ij} |x^*_j| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} |x^*_{t_i}| \right\} \tag{16}$$

It is equal to the optimal value of the objective function of the following linear problem.

$$\beta_i(x^*, \Gamma_i) = max \sum_{j \in J_i} \hat{a}_{ij} |x^*_j| z_{ij} \tag{17}$$

S.t.

$$\sum_{j \in J_i} z_{ij} \leq \Gamma_i$$

$$0 \leq z_{ij} \leq 1 \qquad \forall i, j \in J_i$$

The proof of the above theorem is given in the paper by Bertsimas et al. [27]. By replacing the dual of (4) in the main robust counterpart, it can be formulated as follows:

$$min \quad c^T x + z_0 \Gamma_0 + \sum_{j \in J_0} p_{0j} \tag{18}$$

S.t.

$$\sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{j\in J_i} p_{ij} \le b_i \qquad \forall i$$
$$z_0 + p_{0j} \ge d_j y_j \qquad \forall i \in J_0$$
$$z_i + p_{ij} \ge \hat{a}_{ij} y_j \qquad \forall i \ne 0, j \in J_i$$
$$p_{ij} \ge 0 \qquad \forall i, j \in J_i$$
$$y_j \ge 0 \qquad \forall j$$
$$z_i \ge 0 \qquad \forall i$$
$$-y_j \le x_j \le y_j \qquad \forall j$$
$$l_j \le x_j \le u_j \qquad \forall j$$

Since the amount of demand for each route (d_{imk}) is naturally uncertain in the proposed mathematical model, an uncertainty interval is defined as ($\left[\bar{d}_{imk} - \hat{d}_{imk}, \bar{d}_{imk} + \hat{d}_{imk}\right]$) based on the Bertsimas and Sim approach. According to the interval uncertainty space, each of the uncertain d_{imk} is in a symmetric, finite space and with the center $\bar{d}_{imk}$ in the form of $\hat{d}_{imk} = \rho \times \bar{d}_{imk}$. Where $\bar{d}_{imk}$ the estimated value of the customer demand parameter is, $\hat{d}_{imk}$ is the fluctuation rate of the demand parameter and $\rho > 0$ is the level of uncertainty.

According to the proposed mathematical model, the constraint (6) due to the uncertain parameter's existence leads to uncertainty of the model; hence, it should be made robust based on Bertsimas et al. [27] proposed model. Due to the implemented changes, the demand-related constraints are rewritten as follows.

$$\bar{d}_{imk} - \hat{d}_{imk} \le x_{imk} \le \bar{d}_{imk} + \hat{d}_{imk} \qquad \forall i, m = 1,2,...,N, \qquad \forall k = 1,2,...,K \tag{19}$$

It should be noted that the levels of conservatism (uncertainty) associated with constraint (18) are equal to $\Gamma_{\text{crt}} \in [0, 1]$, which have similar definitions to the proposed Bertsimas et al. [27] model. The final model of the robust problem is formulated by substituting equation (19) instead of equation (6), as follows.

(1-5)
$$\bar{d}_{imk} - \hat{d}_{imk} \le x_{imk} \le \bar{d}_{imk} + \hat{d}_{imk} \qquad \forall i, m = 1,2,...,N, \qquad \forall k = 1,2,...,K \tag{19}$$
(7-13)

5. Experimental results

All variables should be italic throughout the text.

In this section, the proposed robust optimization model is investigated. For this purpose, a real-world case study has been designed and implemented in GAMS software. In this optimization problem, Qeshm Airport is considered the central hub. Six airports of Tehran, Mashhad, Isfahan, Shiraz, Bandar Abbas, and Kish are considered as demand and candidate points for the hub. Among these points, two points should be selected as the hub. Table 2 shows the distance between points in kilometers and minutes. Also, the cost of traveling between different places, the cost of overloading, and ticket prices are reported in Table 3.

Table 2. Distance between points (airports)

Path (From/To)	Distance between hubs and nodes (km)	Distance between hubs and nodes (minutes)
Qeshm/ Tehran	1,072	100
Qeshm/ Mashhad	1,082	115
Qeshm/ Isfahan	775	90
Qeshm/ Shiraz	479	65
Qeshm/ Bandar Abbas	22	15
Qeshm/ Kish	215	45
Tehran/Mashhad	741	90
Tehran/ Isfahan	338	60
Tehran/ Shiraz	682	80
Tehran/ Bandar Abbas	1,050	85
Tehran/ Kish	1,043	108
Mashhad/ Isfahan	833	75
Mashhad/ Shiraz	991	90
Mashhad/ Bandar Abbas	1,060	95
Mashhad/ Kish	1,203	110
Isfahan/ Shiraz	348	56
Isfahan/ Bandar Abbas	750	65
Isfahan/ Kish	714	85
Shiraz/ Bandar Abbas	454	50
Shiraz /Kish	370	55
Bandar Abbas/Kish	234	50

Table 3. Cost of traveling, overloading, and ticket prices

Path (From/To)	Unit cost of traveling between hubs and nodes (Rials/person)	Ticket price between nodes (Rials)	Overloading cost between nodes (Rials)
Qeshm/ Tehran	5,200,000	6,500,000	130,000
Qeshm/ Mashhad	5,720,000	7,150,000	143,000
Qeshm/ Isfahan	3,480,000	4,350,000	87,000
Qeshm/ Shiraz	2,680,000	3,350,000	67,000
Qeshm/ Bandar Abbas	1,200,000	1,500,000	30,000
Qeshm/ Kish	2,320,000	2,900,000	58,000
Tehran/Mashhad	3,960,000	4,950,000	99,000
Tehran/ Isfahan	3,400,000	4,250,000	85,000
Tehran/ Shiraz	3,432,000	4,290,000	85,800
Tehran/ Bandar Abbas	5,200,000	6,500,000	130,000
Tehran/ Kish	5,600,000	7,000,000	140,000
Mashhad/ Isfahan	4,320,000	5,400,000	108,000
Mashhad/ Shiraz	4,416,000	5,520,000	110,400
Mashhad/ Bandar Abbas	5,520,000	6,900,000	138,000
Mashhad/ Kish	5,440,000	6,800,000	136,000
Isfahan/ Shiraz	2,976,000	3,720,000	74,400
Isfahan/ Bandar Abbas	3,040,000	3,800,000	76,000
Isfahan/ Kish	4,720,000	5,900,000	118,000
Shiraz/ Bandar Abbas	2,576,000	3,220,000	64,400
Shiraz /Kish	3,360,000	4,200,000	84,000
Bandar Abbas/Kish	2,120,000	2,650,000	53,000

Finally, Table 4 shows the other input parameters of the case study, including the number of flights, the amount of demand, and the route's capacity.

Table 4. Other input parameters of the problem

Path (From/To)	Number of flights between hubs and nodes (daily)	Flight Class Demand (daily)	Demand for additional flight class load (kg/flight)	Amount of workshop goods (kg)	Path capacity between the central hub and other hubs
Qeshm/ Tehran	11	20	700	1,000	30
Qeshm/ Mashhad	1	2	1,000	800	5
Qeshm/ Isfahan	0.5	1	500	500	2
Qeshm/ Shiraz	1	2	400	400	4
Qeshm/ Bandar Abbas	0.5	1	100	100	2
Qeshm/ Kish	1	2	150	150	4
Tehran/Mashhad	22	30	400	800	50
Tehran/ Isfahan	4	6	300	800	10
Tehran/ Shiraz	16	20	300	800	30
Tehran/ Bandar Abbas	17	25	300	800	40
Tehran/ Kish	30	45	300	800	60
Mashhad/ Isfahan	2	4	400	500	6
Mashhad/ Shiraz	3	5	400	500	8
Mashhad/ Bandar Abbas	2	5	300	700	8
Mashhad/ Kish	9	15	300	650	20
Isfahan/ Shiraz	1	2	200	300	5
Isfahan/ Bandar Abbas	2	5	300	500	8
Isfahan/ Kish	4	6	300	500	9
Shiraz/ Bandar Abbas	3	5	300	300	8
Shiraz /Kish	3	6	200	300	8
Bandar Abbas/Kish	2	4	200	150	7

The proposed robust optimization mathematical planning model based on the above case study was implemented in GAMS software on a 5-core system with a 3 GHz CPU and 2GB RAM and solved using CPLEX solver. The obtained results for the case study are reported in detail in Table 5.

Table 5. Optimization results for the case study

Case Study	**The value of the first objective function (Z_1)**	**The value of the second objective function (Z_2)**	**Solution time (Sec)**
Value	323786435000	202253786000	493.2

The effect of important parameters of the problem has also been investigated. First, the effect of demand on the objective function of the problem was evaluated. As shown in Figure 2, the growth in demand directly links to the increase in the value of objective functions. In fact, the revenue and cost of the whole network increased as the number of passengers increased.

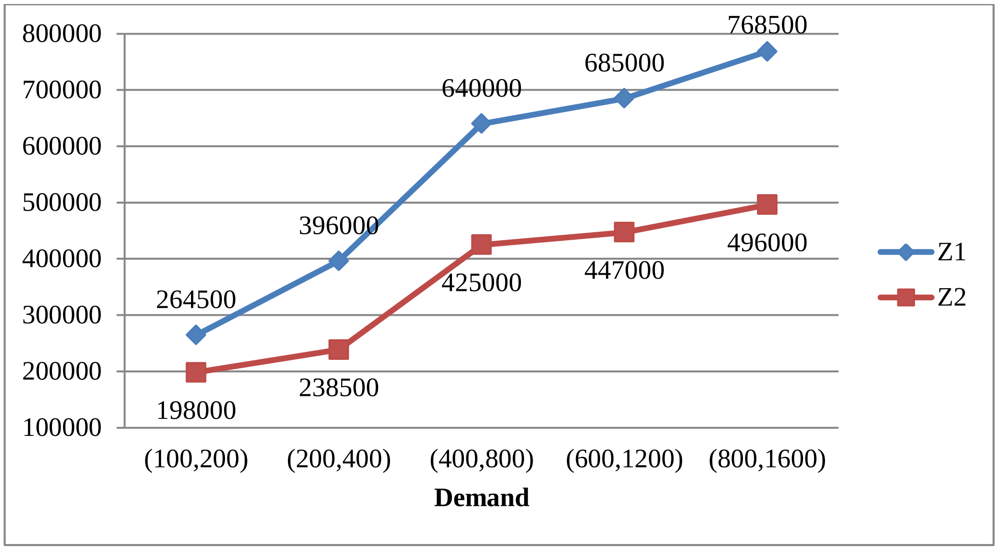

Figure 2. Comparison of solution time

Also, the effect of route capacity on the objective function was analyzed. For this purpose, the capacity has changed between -10 to +10 percent. According to Figure 3, the capacity of the route has only affected the total cost, so that as the capacity of the route increases, the total cost decreases.

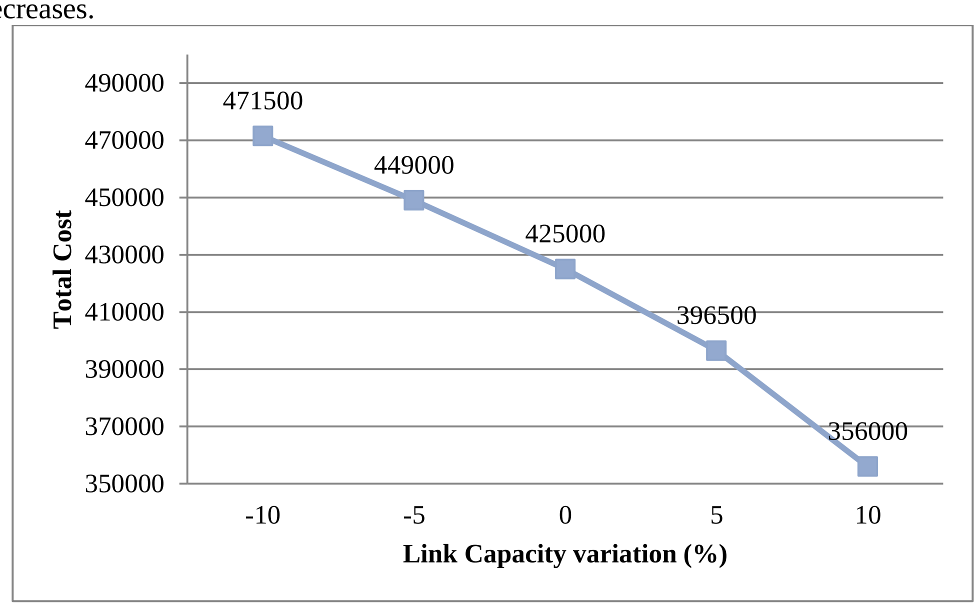

Figure 3. Effect of route capacity on the total cost

6. Conclusion

In this study, a mathematical planning model under uncertainty of the hub location and revenue management problem is developed using a robust optimization approach. First, to validate the proposed mathematical model, a real-world case study with a central hub and six airports is presented and solved using the CPLEX solver in GAMS software. The results of the problems and the values of the objective functions are then reported. Finally, a sensitivity analysis was performed on the key parameters of the problem, and their effects on the objective functions of the problem were investigated. The proposed model help manager to design the network among nodes considering several hub nodes that makes minimum cost while maximum service level achieved. Specially in aviation industry where the setup and operating costs of airplane are very high.

As mentioned previously, the revenue objectives of this study included ticket sales, overload, and workshop load, while other objectives such as revenue from ticket refunds by passengers, revenue from price differences due to changes in flight dates could be considered. In this study, Bertsimas et al. [27] approach has been used for robust model optimization. However, other approaches such as the worst-case scenario can be used. Moreover, other approaches such as Fuzzy theory or stochastic programming can be applied to consider uncertainty. Furthermore, to solve the problem in large dimensions, meta-heuristic algorithms such as genetic algorithm (GA), particle swarm optimization (PSO) algorithm, ant colony optimization (ACO) algorithm, and gray wolf optimization (GWO) algorithm can be used.

References

[1] Sierag DD et al (2015) Revenue management under customer choice behaviour with cancellations and over-booking. Eur J Oper Res 246(1):170–185.

[2] Çetiner D (2013) Fair revenue sharing mechanisms for strategic passenger airline alliances, vol 668. Springer, Berlin

[3] Tikani, H., Honarvar, M., & Mehrjerdi, Y. Z. (2018). Developing an integrated hub location and revenue management model considering multi-classes of customers in the airline industry. Computational and Applied Mathematics, 37(3), 3334-3364.

[4] Yaman H (2008) Star p-hub median problem with modular arc capacities. Comput Oper Res 35(9):3009–3019.

[5] Mohammadi, M., Tavakkoli-Moghaddam, R., 2015. Design of a fuzzy bi-objective reliable *p*-hub center problem. Journal of Intelligent & Fuzzy Systems, (Preprint), 1-18. DOI: 10.3233/IFS-151846.

[6] Balin, S. (2011). Parallel machine scheduling with fuzzy processing times using a robust genetic algorithm and simulation. *Information Sciences*, *181*(17), 3551-3569.

[7] Behnamian, J., & Ghomi, S. F. (2014). Multi-objective fuzzy multiprocessor flowshop scheduling. *Applied soft computing*, *21*, 139-148.

[8] Slowinski, R., and Hapke, M. (2000). *Scheduling Under Fuzziness*, Physica-Verlag Editions, New York.

[9] Parvaresh, F., Moattar Husseini, S.M., Hashemi Golpayegany, S.A., Karimi, B., 2012. Hub network design problem in the presence of disruptions. Journal of Intelligent Manufacturing. DOI 10.1007/s10845-012-0717-7.

[10] Yaman H, Elloumi S (2012) Star p-hub center problem and star p-hub median problem with bounded path lengths. Comput Oper Res 39(11):2725–2732.

[11] Xiao Y, Fu X, Zhang A (2013) Demand uncertainty and airport capacity choice. Transp Res Part B Methodol 57:91-104.

[12] Azizi, N., et al. (2014). "The impact of hub failure in hub-and-spoke networks: Mathematical formulations and solution techniques." Computers & Operations Research.

[13] Hörhammer, A. M. (2014, January). Dynamic hub location problems with single allocation and multiple capacity levels. In 2014 47th Hawaii International Conference on System Sciences (pp. 994-1003). IEEE.

[14] Adibi A, Razmi J (2015) 2-Stage stochastic programming approach for hub location problem under uncertainty: a case study of air network of Iran. J Air Transp Manag 47:172–178

[15] Damgacioglu H et al (2015) A genetic algorithm for the un capacitated single allocation planar hub location problem. Comput Oper Res 62:224–236.

[16] Grauberger, W., & Kimms, A. (2016). Airline revenue management games with simultaneous price and quantity competition. Computers & Operations Research, 75, 64-75.

[17] He, H. Z. (2016). Structural properties in a hub-to-hub network revenue management problem. Journal of the Operations Research Society of China, 4(4), 503-516.
[18] Alumur, S. A., Nickel, S., Saldanha-da-Gama, F., & Seçerdin, Y. (2016). Multi-period hub network design problems with modular capacities. Annals of Operations Research, 246(1-2), 289-312.
[19] Alumur, S. A., Nickel, S., Rohrbeck, B., & Saldanha-da-Gama, F. (2018). Modeling congestion and service time in hub location problems. Applied Mathematical Modelling, 55, 13-32.
[20] Huo, J. Z., Hou, Y. T., Chu, F., & He, J. K. (2019). A Combined Average-Case and Worst-Case Analysis for an Integrated Hub Location and Revenue Management Problem. Discrete Dynamics in Nature and Society, 2019.
[21] Salehi, M., & Tikani, H. (2020). Using revenue management technique to allocate the capacity in reliable hub network design under uncertain air passenger traffic. *Journal of Industrial Engineering and Management Studies*, *7*(2), 139-164.
[22] Čvokić, D. D., & Stanimirović, Z. (2020). A single allocation hub location and pricing problem. *Computational and Applied Mathematics*, *39*(1), 1-24.
[23] Rahmati, R., & Neghabi, H. (2021). Adjustable robust balanced hub location problem with uncertain transportation cost. *Computational and Applied Mathematics*, *40*(1), 1-28.
[24] Tiwari, R., Jayaswal, S., & Sinha, A. (2021). Alternate solution approaches for competitive hub location problems. *European Journal of Operational Research*, *290*(1), 68-80.
[25] Willey, L. C., & Salmon, J. L. (2021). A method for urban air mobility network design using hub location and subgraph isomorphism. *Transportation Research Part C: Emerging Technologies*, *125*, 102997.
[26] Pishvaee, M. S., Torabi, S. A., & Razmi, J. (2012). Credibility-based fuzzy mathematical programming model for green logistics design under uncertainty. Computers & Industrial Engineering, 62(2), 624-632.
[27] Bertsimas, D., Pachamanova, D., & Sim, M. (2004). Robust linear optimization under general norms. Operations Research Letters, 32(6), 510-516.