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Evaluation of general two-stage network systems in the presence of undesirable and non-discretionary data

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Data Envelopment Analysis is one of the most appropriate methods in Evaluation of decision-making units in the real world. That is why researchers have always tried to improve and develop existing methods and approaches in this field. Network Data Envelopment Analysis is used to evaluate the efficiency of network systems by considering processes within divisions. In the evaluation of network systems, one of the challenges is the presence of undesirable and non-discretionary data in the system. Not many conducted have been done about the simultaneous presence of these factors in general two-stage network systems. For this reason, by extending CCR model and combining some methods in this study, we presented a model that is able to evaluate two-stage systems with the mentioned conditions. One of the strengths of the proposed model in this study is the achievement of the efficiency of the system and divisions simultaneously. At the end of the article, we analyzed the results with a numerical example. The results show the ability of the presented model in evaluating the systems under investigation.

Keywords: *Network data envelopment analysis, Evaluation, Efficiency, Non-discretionary data, Undesirable data.*

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1. Introduction

In a production system, the input usually becomes an output after going through several processes. Traditional data envelopment analysis (DEA) models consider the system as a closed unit, disregarding the processes in the divisions. However, since a unit may be composed of several divisions operating interdependently, ignoring the operations of these divisions may obtain incorrect results. This idea was discussed by Charnes et al. in 1986, which found that army recruitment actually had two stages, creating awareness through advertising and signing contracts [1].

Separation of large operations into smaller parts makes the efficiency score more realistic and helps us identify the real effects of factors. For this reason, Fare and Grosskopf proposed the idea of network data envelopment Analysis (NDEA) in 2000, taking the operation of component processes into consideration in calculating the efficiency of the system. They considered internal structure of

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the system being evaluated [2]. This approach examines a sequence of processes in different division of the systems. NDEA is able to identify divisions that contribute to inefficiencies in systems.

In many real-world issues that are examined using DEA, data is not always normal. Sometimes issues include special data that we have to deal with in their own ways. Some special data without physical value, such as fuzzy, Stochastic, interval, ordinal data, etc., can be present in the production process. Many studies have been done on these data with the DEA approach by researchers. Fallah et al. studied on Discriminant Analysis and Data Envelopment Analysis with Specific Data (2020) [3]. Pourmahmoud and Bafekr Sharak, (2020) employed a fuzzy data envelopment analysis to measure cost efficiency of DMUs [4]. Pourmahmoud and norouzi (2022) Provided a new model for evaluating and ranking DMUs with ordinal data [5].

Furthermore, in real word issues, data is not always desirable or discretionary. Sometimes non-discretionary or undesirable data, or both, are present in the system. Undesirable outputs refer to the data in which a greater amount being produced is less desirable and undesirable inputs refer to the data in which a less amount being used is less desirable. Non-discretionary data refer to the data whose values are fixed and cannot be changed by the administrator. In the following, we will provide explanations about each of the undesirable and non-discretionary factors, and two-stage network systems.

1.1 Undesirable data

In some systems, the production process creates the products we need. However, sometimes products that we do not need, such as environmental pollution resulting from economic activities, are emitted from the production process. These unwanted products are mentioned as undesirable output. Traditional DEA methods improve efficiency of units by reducing input or expanding output. But reduced input and expanded output also include undesired data [6]. Therefore, these methods ignore the undesirable data and incorrect results may be achieved during calculations. Several approaches for dealing with undesirable factors are introduced, including data transformation, input-output exchange, slacks-based measures and weak disposability [7].

Undesirable outputs were proposed by Pittman in 1983 [8]. After that many researchers studied on this type of outputs, Sifford and Zhou (2002) presented a model with desirable and undesirable data based on BCC model, in which the undesirable outputs were multiplied by a negative [9]. The challenge of this model was investigated by Fare and Grosskopf that was obtaining different answers, which was accepted by Sifford and Zhou. They solved this problem by defining a directed distance function in 2004 [10]. Jahanshahloo et al. (2004) presented multi-objective linear programming to solve problems with undesirable data [11]. KordRostami and Amirteimoori (2005) presented a multi-stage model in which undesirable variables with a negative sign were used in the calculation of weights [12]. Amirteimoori et al. (2006) used a model with the aim of improving efficiency by increasing undesirable inputs and reducing undesirable outputs [13]. Akhtar et al. (2013) presented a model to minimize undesirable and maximize desirable outputs [14]. Homayounfar and Amirteimoori (2016) used a fuzzy network method based on DEA in the presence of desirable and undesirable outputs in their study [15]. Madadi et al. (2018) expanded a resource allocation model for evaluation of 25 branches of an Iranian Tejarat bank in the presence of undesirable data [16]. Seihani Parashkouh et al. (2020) proposed two non-linear technologies based on weak disposability definitions for two stage systems with undesirable data [17]. Omrani et al. (2022) developed NDEA model with negative inputs and undesirable outputs [18]. Azizi and Shirvani presented a model by Developing a Two-Stage Network Data Envelopment Analysis Model with Desirable and Undesirable Outputs (2022) [19].

Other studies in this field include Lovell and Pasteur in 1995 [20], Pastor in 1996 [21], Deason et al. in 2001[22], Zhu et al. in 2008 [23], Sahoo et al. in 2011 [24], You and Yan in 2011 [25], Song et al. in 2012 [26], Wang et al. in 2012 [27], Leleu in 2013 [28], Sueyoshi and Wang in 2014 [29].

1.2 Non-discretionary data

One of the advantages of the DEA approach is the identification of targets for inefficient DMUs to become efficient. This is based on the reduction of the inputs and expansion of the outputs. When some of the inputs or outputs are non-discretionary, this approach is not useful and yields incorrect efficiency scores.

The first study on non-discretionary data was performed by Bunker and Murray in 1986 [30]. Their model evaluates units by comparing them in more stringent environments in terms of non-discretionary factors. Their other model (1986), which is based on the idea of discretionary or non-discretionary condition of data, is currently one of the most widely used models in this field [31]. In 1991, using regression analysis, Ray investigated the effect of non-discretionary factors as independent variables on unit efficiency [32]. In 1997, Ruggiero presented a model that selects the reference set from units with a stringent environment or at least a similar environment in the presence of non-discretionary data [33]. Hosseinzadeh et al. (2007) used the super efficiency approach in DEA in the presence of non-discretionary inputs [34]. Jahanshahloo et al. used a non-radial DEA to discuss non-discretionary data in 2007 [35]. Camanho et al. (2009) presented a model that treats non-discretionary data depending on their classification as internal or external [36]. Gholam Abri and Fallah Jelodar (2012) proposed a linear model by considering non-discretionary factors and review on previous models [37]. Several simulation studies have been conducted to investigate the effect of the non-discretionary factors on efficiency score such as Yu (1998) [38], Syrjanen (2004) [39], Muniz et al. (2006) [40].

1.3 Two-Stage Systems

Many systems have a multi-stage structure, where basic material goes through a number of stations to become the final outputs. In this system, a stage may have several divisions connected in different structures. The system that are reviewed in the conventional network DEA is a system composed of a number of divisions connected in series, with only one division in each stage. The simplest type of these systems is the basic series structure with two stages, where all of the outputs of the first division are consumed as input in the second division. In this type, the first division has an exogenous input and the second division has a final output only. These systems are known as basic two-stage systems. The structure of the basic two-stage system is shown in Figure 1[41].

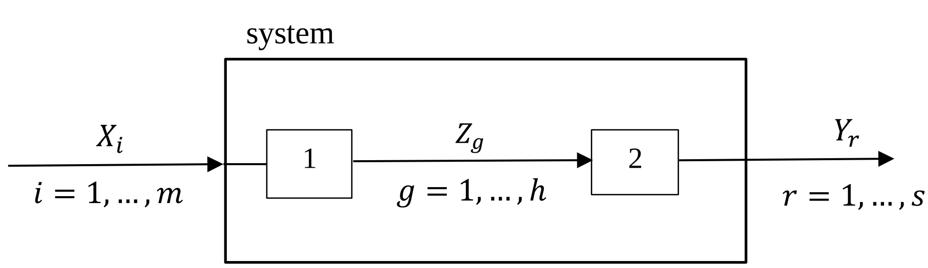

Figure 1. Structure of the basic two-stage system

In two-stage network systems some intermediate products may come out of the system, and the second division may need exogenous input to become the final product. In this case, we have a general two-stage system, which allows the first division to have final outputs and the second division to have exogenous inputs [6]. The structure of the general series system will be reviewed in this study, as shown in Figure 2. We will discuss the efficiency of this structure in the simultaneous presence of the undesirable and non-discretionary data.

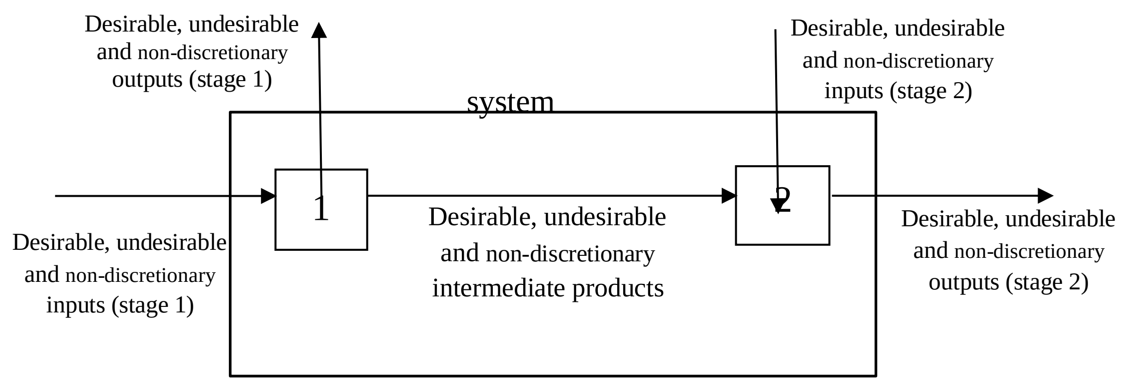

Figure 2. Structure of the two-stage system under review

Several studies have been done on NDEA deal with two-stage systems that applied to measure the efficiency of these systems. Fare and Whittaker (1995) formulated the first distance function model to measure the system efficiency of the general two-stage system [42]. Seiford and Zhu (1999) calculated the efficiency of commercial banks of United states [43]. Zhu (2000) evaluated 500 companies with two-stage structure [44]. Fare (2000) presented a method by considering intermediate products to calculate efficiency of the systems [45]. Kao and Hwang (2009) presented a relational model to calculate Efficiency decomposition in NDEA [46]. Kao (2014) proposed a general slack base model for evaluating the efficiency of the systems [47]. Moradi et al. (2021) presented a method based on the fuzzy interpretive structural modeling (FISM) to find a common set of weights (CSWs) for the variables involved [48]. Nasseri et al. propose a two-phase approach to solve Fuzzy Flexible Linear programming [49].

These studies on NDEA are considerable, but not many studies have been done on Two-Stage Systems with undesirable and non-discretionary data. So, in this study we will evaluate these systems. We will evaluate the systems by defining different distance parameters for divisions of the systems.

In the following, to evaluate the mentioned systems, we will first describe the basic model and present proposed model. To review the performance of the proposed model, a numerical example is presented and the results will be interpreted in detail.

2. Basic model

In the evaluation of decision-making units with a radial model, outputs are kept constant at their level and inputs are reduced, or the inputs are kept constant at their level and outputs are increased. The envelopment form of input-oriented basic model reduces the inputs by decreasing θ. This model, which was presented by Charnes et al. under the Constant Returns to Scale technology, has the following form [6]:

$$E_o = \theta - \varepsilon(\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+})$$

s.t.

$$\begin{aligned}
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-} = \theta x_{io} && i = 1, \dots, m \qquad (1)\\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^{+} = y_{ro} && r = 1, \dots, s\\
&\lambda_j, s_i^{-}, s_r^{+} \geq 0, \quad j = 1, \dots, m \quad, i = 1, \dots, m \quad, \quad r = 1, \dots, s
\end{aligned}$$

Model (1), which is used to evaluate homogeneous decision-making units, is considered a radial model, where E_o is the efficiency of the DMU being evaluated, and it is efficient only if $\theta = 1 \; and \; s_i^{-}, \; s_r^{+} = 0, \; i = 1, \dots, m, \quad r = 1, \dots, s.$

As mentioned in 1.3., (Two-Stage Systems) Fare and Whittaker (1995) proposed the first distance function model to evaluate the general two-stage system. The basic idea is convexity of the production possibility set and strong disposability. Under constant returns to scale, the model from the input side is:

$$E_o = \min \theta$$

s.t.

$$\begin{aligned}
&\sum_{j=1}^{n} \lambda_j^{(1)} x_{ij}^{(1)} \leq \theta x_{io}^{(1)} && i = 1, \dots, m_1\\
&\sum_{j=1}^{n} \lambda_j^{(2)} x_{ij}^{(2)} \leq \theta x_{io}^{(2)} && i = m_1 + 1, \dots, m\\
&\sum_{j=1}^{n} \lambda_j^{(1)} z_{gj} \geq z_{go} && g = 1, \dots, h\\
&\sum_{j=1}^{n} \lambda_j^{(2)} z_{gj} \leq z_{go} && g = 1, \dots, h \qquad (2)\\
&\sum_{j=1}^{n} \lambda_j^{(1)} y_{rj}^{(1)} \geq y_{ro}^{(1)} && r = 1, \dots, s_1\\
&\sum_{j=1}^{n} \lambda_j^{(2)} y_{rj}^{(2)} \geq y_{ro}^{(2)} && r = s_1 + 1, \dots, s\\
&\lambda_j^{(1)}, \lambda_j^{(2)} \geq 0 && j = 1, \dots, n
\end{aligned}$$

In this model, the non-Archimedean number ε has been ignored for simplicity of expression. With variable returns to scale technology $\sum_{j=1}^{n} \lambda_j^{(k)} = 1 \,, \; k = 1, 2,$ are added. The model (2) can be applied when all of the data are desirable and discretionary. If one of these two types or both are present in the system, it will not be implemented. Several studies have been done on two-stage network systems in the presence of undesirable and non-discretionary data, separately. Here we mention some studies that have been done at the presence of undesirable data in two-stage network systems. Fallahi et al. (2011)

presented a model for measuring efficiency and productivity change in power electric generation management companies by using DEA [50]. Maghbouli et al. (2014) presented a model to evaluate Two-stage network structures with undesirable outputs [51]. Mirhedayatian et al. (2014) presented a novel network data envelopment analysis model for evaluating green supply chain management [52]. Fathalikhani proposed a two-stage DEA Model Considering Shared Inputs, Free Intermediate Measures and Undesirable Outputs in 2016 [53].

Some studies have been done at the presence of non-discretionary data in two-stage network systems. Taleb et al. (2018) presented a super efficiency slack-based measure to evaluate two-stage network systems in the presence of non-discretionary factors and mixed integer-valued data envelopment analysis [54]. Barat et al. (2018) applied data envelopment analysis for nonhomogeneous mixed networks [55]. Galagedera proposed a two-stage data envelopment analysis model with non-discretionary first stage output in mutual fund performance appraisal in 2019 [56].

As mentioned, the above studies have been done separately on undesired and non-discretionary data in two-stage network systems. In the simultaneous presence of these data in general two-stage network systems, the aforementioned studies are not responsive, so we will present the proposed model to solve this challenge in section 3.

3. Proposed model

In this section, we present a model for evaluating two-stage general network systems in the simultaneous presence of undesired and non-discretionary data. we will use the input-output exchange approach for undesirable factors and keep constant approach for non-discretionary factors to manage them. We assume n two-stage systems for evaluation. in this case, we will present the proposed model assuming the following assumptions:

Suppose for the division p of system $j,\ \ j=1,2,\dots,n.$:

Number of inputs=m_p ,$p=1,2.$

Number of desirable inputs=d_p ,$p=1,2.$

Number of undesirable inputs= q_p ,$p=1,2.$

Number of non-discretionary inputs=t_p ,$p=1,2.$

Where $d_p+u_p+t_p=m_p,\ \ p=1,2.$

Number of outputs=s_p ,$p=1,2.$

Number of desirable outputs=l_p ,$p=1,2.$

Number of undesirable outputs= t_p ,$p=1,2.$

Number of non-discretionary outputs=u_p ,$p=1,2.$

Where $l_p+t_p+u_p=s_p,\ \ p=1,2.$

Number of intermediate products =$h_p,\ \ p=1,2.$

With the above assumptions, the proposed model is presented as follows:

$$E_o = \min(\omega_1\theta_1 + \omega_2\theta_2) - \varepsilon[\sum_{p=1}^{2}\left(\sum_{i=1}^{d_p} s^-{}_{pi} + \sum_{i=d_p+1}^{d_p+q_p} s^+{}_{pi} + \sum_{i=d_p+q_p+1}^{m_p} s^-{}_{pi} + \sum_{r=1}^{l_p} s^{\circ+}{}_{pr} + \sum_{r=l_p+1}^{l_p+t_p} s^{\circ-}{}_{pr} + \sum_{r=l_p+t_p+1}^{s_p} s^{\circ+}{}_{pr}\right)$$

s.t.

$$1.\ \sum_{j=1}^{n} \lambda_j{}^{(p)} x_{ij}{}^{(p)} + s_{pi}{}^- = \theta_p x_{io}{}^{(p)} \qquad i = 1, \dots, d_p\ , \qquad p = 1,2$$

$$2.\ \sum_{j=1}^{n} \lambda_j{}^{(p)} x_{ij}{}^{(p)} - s_{pi}{}^+ = x_{io}{}^{(p)} \qquad i = d_p + 1, \dots, d_p + q_p, \quad p = 1,2 \qquad (3)$$

$$3.\ \sum_{j=1}^{n} \lambda_j{}^{(p)} x_{ij}{}^{(p)} + s_{pi}{}^- = x_{io}{}^{(p)} \qquad i = d_p + q_p + 1, \dots, m_p\ , \qquad p = 1,2$$

$$4.\ \sum_{j=1}^{n} \lambda_j{}^{(p)} y_{rj}{}^{(p)} - s^{\circ+}{}_{pr} = y_{ro}{}^{(p)} \qquad r = 1, \dots, l_p\ , \qquad p = 1,2$$

$$5.\ \sum_{j=1}^{n} \lambda_j{}^{(p)} y_{rj}{}^{(p)} + s^{\circ-}{}_{pr} = \theta_p y_{ro}{}^{(p)} \qquad r = l_p + 1, \dots, l_p + t_p, \qquad p = 1,2$$

$$6.\ \sum_{j=1}^{n} \lambda_j{}^{(p)} y_{rj}{}^{(p)} - s^{\circ+}{}_{pr} = y_{ro}{}^{(p)} \qquad r = l_p + t_p + 1, \dots, s_p\ , \qquad p = 1,2$$

$$7.\ \sum_{j=1}^{n} \lambda_j{}^{(1)} z_{gj} \geq \sum_{j=1}^{n} \lambda_j{}^{(2)} z_{gj} \qquad g = 1, \dots, h$$

$$\lambda_j{}^{(p)}, s_{pi}{}^-, s_{pi}{}^+, s^{\circ+}{}_{pr}\ , s^{\circ-}{}_{pr} \geq 0, \quad for\ all\ \ p, j\ , i\ , r$$

ω_1 and ω_2 are the weights assigned to each section by the system manager where $\omega_1 + \omega_2$ =1, $\omega_i \geq 0, i = 1,2$. The first six set of constraints (1-6, p=1) in model (2) correspond to the first division of the system being evaluated. The first three constraints of this category (1-3, p=1) are related to the inputs and the second three constraints (4-6, p=1) are related to the outputs of this division. The first constraint of this category (1, p=1) corresponds to the desirable inputs, and the second constraint (2, p=1) corresponds to the undesirable inputs and the third constraint (3, p=1) corresponds to the non-discretionary. The constraints of outputs are also given similarly. The second constraint category corresponds (7) to the intermediate products and the third constraints set (1-6, p=2) corresponds to the second division of the system, like what mentioned for the first division. This model ensures that the intermediate product as an output is greater than or equal to that as an input. The advantage of this model is that it is able to measure the system and division efficiencies at the same time.

E_o is the efficiency score of the system being evaluated and θ_1, θ_2 are the efficiency scores of the division 1,2 respectively.

Definition 3.1. The evaluated system is efficient when the value of the objective function (E_o) is equal to 1, and $s_{1i}{}^- = s_{1i}{}^+ = s^\circ{}_{1r}{}^+ = s^\circ{}_{1r}{}^- = s_{2i}{}^- = s_{2i}{}^+ = s^\circ{}_{2r}{}^+ = s^\circ{}_{2r}{}^- = 0, \quad for\ all\ \ j\ , i\ , r.$

The first division of evaluated system is efficient when θ_1 is equal to 1, and $s_{1i}{}^- = s_{1i}{}^+ = s^\circ{}_{1r}{}^+ = s^\circ{}_{1r}{}^- = 0, \quad for\ all\ \ j\ , i\ , r.$

The second division of evaluated system is efficient when θ_2 is equal to 1, and $s_{2i}{}^- = s_{2i}{}^+ = s^\circ{}_{2r}{}^+ = s^\circ{}_{2r}{}^- = 0, for\ all\ \ j\ , i\ , r.$ Therefore, the system is efficient when both divisions are efficient.

Now with a numerical example, we will examine the results of the model implementation on 5 two-stage network systems.

4. Numerical Example

In this section we consider a simple structural example that undesirable and non-discretionary data are present in two-stage network systems. With these conditions, consider 5 systems A, B, C, D, E with the structure shown in Figure 2 and the data shown in Table 1 and 2:

Table1: Data of the first division of the systems

	$x_1^{(1)}$	$x_2^{(1)}$	$x_3^{(1)}$	$x_4^{(1)}$	$z^{(1)}$	$y_1^{(1)}$	$y_2^{(1)}$	$y_3^{(1)}$	$y_4^{(1)}$
A	1.000	0.700	0.900	2.000	0.500	1.000	0.500	4.000	2.000
B	3.000	2.000	0.500	1.000	5.000	1.000	0.700	3.000	1.000
C	2.000	1.500	1.000	3.000	2.000	2.000	1.000	3.000	4.000
D	0.010	1.000	3.000	5.000	3.000	2.000	9.000	6.000	5.000
E	0.500	0.400	5.000	4.000	1.000	1.000	4.000	0.300	3.000

Table2: Data of the second division of the systems

	$x_1^{(2)}$	$x_2^{(2)}$	$x_3^{(2)}$	$x_4^{(2)}$	$z^{(2)}$	$y_1^{(2)}$	$y_2^{(2)}$	$y_3^{(2)}$	$y_4^{(2)}$
A	0.500	0.400	0.800	1.000	0.400	1.500	1.000	5.000	4.000
B	1.000	3.000	0.400	2.000	3.000	0.800	0.400	6.000	5.000
C	3.000	2.000	0.900	5.000	2.000	3.000	0.800	3.000	2.000
D	4.000	1.000	2.000	4.000	1.000	2.000	3.000	1.000	0.900
E	2.000	0.700	6.000	3.000	0.500	3.000	5.000	0.200	1.000

Data of the first division of the systems are shown in table 1 and Data of the second division of the systems are shown in table 2. The first two inputs are assumed desirable, the third input is undesirable and the fourth input is assumed to be non-discretionary, in both divisions of the systems. Every system has an intermediate product. Also, the first two outputs are assumed to be desirable, the third outputs are undesirable and the fourth outputs is assumed to be non-discretionary, in both division of the systems. Model (3) is applied on data of Tables 1 and 2 and the results are given in Tables 3, 4 and 5:

Table3: results of model (3) – division 1

	θ_1	${s_{11}}^-$	${s_{12}}^-$	${s_{13}}^+$	${s_{14}}^-$	${s^\circ_{11}}^+$	${s^\circ_{12}}^+$	${s^\circ_{13}}^-$	${s^\circ_{14}}^+$
A	0.892857	0.390357	0.000000	0.100000	0.000000	0.000000	2.000000	1.321429	0.250000
B	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
C	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
D	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
E	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table4: results of model (3) – division 2

	θ_2	${s_{21}}^-$	${s_{22}}^-$	${s_{23}}^+$	${s_{24}}^-$	${s^\circ_{21}}^+$	${s^\circ_{22}}^+$	${s^\circ_{23}}^-$	${s^\circ_{24}}^+$
A	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
B	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
C	0.611111	0.000000	0.505555	4.366666	2.166666	0.000000	3.700000	0.00000	0.166667
D	0.470000	0.580000	0.000000	1.853333	2.033333	0.000000	0.233333	0.010000	0.000000
E	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table5: results of model (3) – system

	$\boldsymbol{E_o}$
A	0.9464245
B	1.0000000
C	0.8055446
D	0.7349953
E	1.0000000

It should be mentioned that in this example, the values of the weights are considered as $\omega_1 = \omega_2 = \frac{1}{2}$. Data in table 3 shows the results of model (3) applied on first division of the systems, as it can be seen the values of θ_1 for all systems are equal to 1 and ${s_{1i}}^- = {s_{1i}}^+ = {s^\circ_{1r}}^+ = {s^\circ_{1r}}^- =$

$0, \quad for\ all \quad j, i, r$ except system A. Therefore, the first divisions of all systems are efficient except system A.

Table 4 shows the results of the model (3) applied on second division of the system. The values of θ_2 for A, B, E are equal to 1 and $s_{1i}^{-} = s_{1i}^{+} = s^{\circ}{}_{1r}^{+} = s^{\circ}{}_{1r}^{-} = 0, \quad for\ all \quad j, i, r,$ which correspond to these systems. Therefore, the second divisions of A, B, E are efficient.

Data in table 5 shows efficiency score of the system being evaluated, as we expected B and E are efficient, because only in these two systems, both divisions are efficient.

5. Conclusion

In this study, we tried the extension of CCR model to evaluate two-stage network systems in the presence of undesirable and non-discretionary data. The advantage of proposed model is that it is able to measure the system and division efficiencies at the same time. Model (3) is an input-oriented model that can be presented in an output-oriented or input-output form. In the end, by presenting a numerical example with a limited number of data, we investigated the results of the proposed model on the divisions and systems. We saw that The system is efficient when the value of the objective function is equal to 1, and $s_{1i}^{-} = s_{1i}^{+} = s^{\circ}{}_{1r}^{+} = s^{\circ}{}_{1r}^{-} = s_{2i}^{-} = s_{2i}^{+} = s^{\circ}{}_{2r}^{+} = s^{\circ}{}_{2r}^{-} = 0, \quad for\ all \quad j, i, r,$ for all j ,i ,r. Future researches can include related applications with other types of data such as interval, stochastic, fuzzy, etc., on general multi stage systems.

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