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A novel algorithm to allocate customers and retailers in a closed-loop supply chain under probabilistic demand

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In this paper, a closed-loop supply chain (CLSC) is modeled to consider the demand and location of probabilistic customers in order to determine the optimal location of retailers and their allocation to other facilities. The CLSC structure comprises production centers, retailers' centers, probabilistic customers, and collection and disposal centers. This study examines two strategies for identifying the best retailer locations, focusing on 1) the type of expected movement and 2) expected coverage. To achieve this, a bi-objective nonlinear programming model is proposed. This model simultaneously evaluates Strategies 1 and 2 to determine the superior approach. Based on the chosen strategy, the best allocation is determined using two heuristic algorithms, which then establish the optimal retailer locations. Given that the proposed model is NP-hard, a meta-heuristics (non-dominated sorting genetic) algorithm is utilized for the solution process. The model has been implemented using MATLAB software, and an illustrative example has been solved. The example results have indicated the optimal location for retailers. In this example, movement has been prioritized over coverage, and customers have been suggested to move towards retailers.

Keywords*: Closed-loop supply chain, expected coverage distance, Expected coverage time, NSGA-II*

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1. Introduction

Supply chain network design (SCND) is a crucial strategic decision that has recently received significant research attention. Ramezani,et al [1]. In recent years, there has been a significant increase in interest in reverse logistics (RL) and closed-loop supply chains (CLSCs) due to the growing environmental issues. Traditional forward supply chains involve the forward movement of products from suppliers, plants, and distributors to customers. RL, on the other hand, focuses on the reverse flow of products from end customers and includes activities such as collecting, inspecting, repairing, disassembly, disposal, recycling, and remanufacturing of products Biçe and Batun [2], Farrokh,et al [3]. Increasing environmental awareness, social responsibility, and government regulations have

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driven many companies to use recycled products and recover waste Giri and Sharma [4]. Even during and after a critical situation (such as the COVID-19 pandemic), it is crucial to activate both the open- and closed-loop systems within an effective and resilient supply chain network. Mondal and Roy [5].

In this regard, many large companies such as Xerox, Canon, Kodak, Dell, and Acer have put efforts into green operations. For instance, Dell has recently changed their business approach to zero-waste manufacturing and renewable-energy usage Chen and Ulya [6]. Nowadays, (CLSC) systems have become increasingly complex and dynamic with extensive geographical coverage. As a result, (CLSC) is susceptible to a wide range of uncertainties, some of which may lead to disruptions in the (CLSC) Tolooie,et al [7]. When designing a supply chain network (SCN), it is important to consider that SC breakdowns are unexpected and irregular occurrences that disrupt the regular flow of goods and supplies in the chain. As a result, companies in the SC are vulnerable to commercial and operational risks Ghomi-Avili,et al [8]. as disruptions in infrastructure have significant impacts on the operation and performance of the SC. Yavari and Zaker [9]. Therefore, the strategy must consider these contradictions and the need to choose between competitive criteria such as speed, efficiency, quality, cost, and satisfaction, or mixed models. Ebrahimiarjestan and Wang [10].

Allocating retailers in a closed-loop supply chain structure includes production centers, retailer's centers, probabilistic customers, collection, and disposal centers. Comparing the type of movement and possible coverage and use of innovative algorithms in the allocation is a topic that has been less addressed in recent articles. Mondal and Roy [5],Yavari and Zaker [9], Gholizadeh and Fazlollahtabar [11],Yu and Solvang [12], Sazvar,et al [13], Chai,et al [14], Saha,et al [15], Ke and Cai [16].

2. Literature Review

In this section, we present recent studies on the location-routing-inventory problem, closed-loop supply chain, and their synthesis to demonstrate the need for this research. This paper reviews the related literature in two research domains of CLSC under certain and uncertain conditions. Sun,et al [17] Studied the equilibrium of the CLSCN, considering two types of suppliers, the manufacturer's risk aversion, and the suppliers' capacity constraints. this model after The analysis explores the interaction and influence of these factors on the equilibrium, offering a deeper understanding of the system's dynamics. Zhang and Ren [18] presented a CLSC model that involved an original manufacturer, third-party remanufacturer, and retailer. The model stipulated that the remanufacturer could only recycle and remanufacture patented products with patent licensing from the original manufacturer. Newly manufactured and remanufactured products are then sold together in the same market at different prices. Shi,et al [19] developed a multi-objective Mixed Integer Programming Model for a CLSCN design problem. The suggested model optimized overall carbon emissions and network responsiveness in addition to overall costs. An improved genetic algorithm based on the framework of NSGA II was developed in this study to solve the problem and obtain Pareto-optimal solutions. Ahmadi and Amin [20] also developed a multi-period, multi-product, multi-echelon, and multi-customer CLSCN for a mobile phone network, taking into account various types of product returns. The researchers Chen and Chi [21] examined a two-echelon SC with one manufacturer and one retailer and three different reverse channel formats. The purpose of the study was to analyze the impact of the reverse channel structure on the wholesale price, retail price, collection rate, and total channel profits. Mondal and Giri [22] developed a two-period closed-loop green supply chain (CLGSC) model with a single manufacturer and a single retailer to study the effects of green innovation, marketing effort, and collection rate of used products on the SC decisions. Wu,et al [23] designed a multi-objective optimization model for the product family. Next, they formulated CLSC based on a cooperative game model to minimize the manufacturer's total cost and maximize suppliers' total payoffs. In Xiao,et al [24] a dual-channel CLSC structure was proposed where a manufacturer sells to a retailer through both traditional retail and a direct online channel. The components of this model operate in a Stackelberg game. In this setup, the manufacturer acts as the leading channel, owning both the traditional retail channel and an Internet-based direct channel, while the retailer follows as a channel and sells products in the traditional retail channel. in Zheng,et al [25] the authors introduced a CLSC model with dual competitive sales channels. They considered three reverse channel structures in the CLSC Manufacturer collecting (Model M), retailer collecting (Model R), and third-party collecting (Model C) structures. In addition, they showed that a simple price contract consisting of the wholesale price, direct channel price, and transfer price of the used product (in Model R and Model C), with a complementary profit-sharing mechanism, can effectively coordinate dual-channel CLSCs under different recycling channel structures. In Chan,et al [26] a dynamic equilibrium model of oligopolistic CLSCN was created to address the seasonal nature of demand. In this model, demands and returns are uncertain and time-dependent. Additionally, the dynamic Cournot-Nash equilibrium of the oligopolistic network is established through evolutionary variation inequality and projected dynamical systems. Fathollahi-Fard,et al [27] proposed a new mathematical formulation for a multi-objective stochastic CLSCN that takes social impacts into account. Three novel hybrid meta-heuristics were applied to a strategically CLSC based on the proposed model. The researchers simultaneously considered economic and social aspects using specific suppositions. In Kim,et al [28] developed a deterministic mixed-integer optimization model and robust counterparts to address uncertainty in the fashion industry regarding recycled products and customer demand. The researchers demonstrated that a robust counterpart with a budget of uncertainty is equivalent to a robust counterpart with a box uncertainty under specific conditions. In Ma and Li [29] the study focused on the design of a closed-loop supply chain network for hazardous products (HP-CLSCND), encompassing both forward SC and reverse SC. They demonstrated that the inherent uncertainty in CLSCN significantly impacts the overall performance of the network's design. The model addresses the HP-CLSCND problem with uncertain demands and returns. They proposed a two-stage stochastic programming model (scenario-based) to solve this issue, simultaneously considering risk restriction constraints and reward-penalty mechanisms. The primary objective of this model is to minimize the total system cost by making ordering decisions. In Zeballos,et al [30] a two-stage mixed-integer problem (MIP) model was proposed that combines the conditional value at risk and the structure of a CLSCN. In this model, end-customer areas are divided into two parts, namely, the primary market and the secondary market. This approach can prevent a distinct impact on economic performance via changes in quality. In Vahdani and Ahmadzadeh [31] a mixed-integer nonlinear programming (MILP) model was proposed to integrate pricing with facility location and inventory control decisions in a CLSCN in the information and communications technology (ICT) industry. This model maximizes the total profit obtained by selling the new ICT products or collecting the used ICT products. Abdi,et al [32] suggested a new nature-inspired algorithm, the Whale Optimal Station Algorithm (WOA), in a CLSC model. They also utilized the popular algorithm Particle Swarm Optimization (PSO) to address this issue. Additionally, they employed the well-known meta-heuristics Genetic Algorithm (GA) and Simulated Annealing (SA). The researchers used various evaluation metrics to assess the quality of the algorithms' Pareto optimal fronts and conducted a comparative study. Fathollahi-Fard,et al [33] demonstrated the development of a comprehensive CLSC and the design of a network that takes into account both the cost objective and the service efficiency objective of warehouses/hybrid facilities. The proposed MILP model aids decision-making in the facility location and distribution planning in a CLSCN under the two objectives. In another study Kalantari Khalil Abad and Pasandideh [34] a two-stage stochastic programming model was introduced for designing a green CLSC. This model provides an upper limit on emission capacity, which assists governments and industries in managing greenhouse gas emissions. The researchers in Gholizadeh and Fazlollahtabar [11] examined a CLSC with various grades derived from a reverse flow melting process for demand planning to address model uncertainty. The focus of the modeling was on maximizing profitability in the face of demand uncertainty. They explored several aspects of this area and employed a robust and modified GA for optimization. In Yu and Solvang [12] a novel fuzzy-stochastic multi-objective mathematical model was introduced for sustainable CLSCN design. The model aims to balance the trade-off between cost-effectiveness and environmental performance under various uncertainties and demands. The environmental performance of the CLSCN design is assessed based on carbon emissions. Additionally, the model incorporates network flexibility in the decision-making process to ensure that customer demands can be met through different methods.in Alinezahd [35] proposed a Closed-Loop Supply Chain Network (CLSCN) with four echelons in the forward chain (suppliers, plants, distribution centers, and customers) and three echelons in the backward chain (collection centers, inspection centers, and disposal centers). This model addresses a multi-product and multi-period mixed-integer linear programming problem, aiming to maximize profit in the closed-loop supply chain network. Xu,et al [36] proposed a two-stage stochastic model to design the CLSC under a carbon trading scheme in the multi-period planning context by considering the uncertain demands and carbon prices. this model provided a four-step solution procedure with scenario reduction that enables the proposed model to be solved using popular commercial solvers efficiently. Govindan,et al [37] proposed a model that addresses the location-inventory-routing problem by structuring the network. It incorporates a carbon tax policy and a vehicle scheduling problem to minimize emissions and reduce vehicle waiting time, respectively. The approach utilizes stochastic scenarios to handle demand uncertainty and employs an augmented epsilon-constraint method to solve the proposed bi-objective model. The researchers in Mondal and Giri [38] investigated a (CLSC) involving an environmentally conscious manufacturer, a retailer, and a third-party collector while considering government intervention. The manufacturer provides a return policy for defective products within a specific timeframe, and the third-party collector offers consumers an acquisition price for returning their used products. Geon Kim,et al [39] suggested a robust optimization model for a CLSC, considering uncertain demand and time-series patterns of uncertain carbon tax rates based on historical data. This model incorporated the first-order autoregressive model in a set-based robust optimization model to attain a less conservative solution. Furthermore, two new uncertainty sets have been formulated to mirror the time-series pattern using historical data and their manageable robust counterparts.in Zadeh,et al [40] a multi-objective mixed-integer linear programming model was developed to design a green multi-echelon closed-loop supply chain network under uncertainty. The model addresses partial disruptions at distribution centers using a fuzzy credibility constraint approach. Additionally, the ε-constraint method is presented to solve and validate the model in small-sized instances. Furthermore, a Non-dominated Sorting Genetic Algorithm is developed to solve large-sized problems. Researchers in Babaei,et al [41] proposed a branch and bound algorithm in (CLSC), which includes an optimization model and an evaluator model. The model aims to minimize the total supply chain cost, maximize the sustainability score, and minimize inequity among customers simultaneously. To account for real-world conditions, parameters related to labor and demand are assumed under uncertainty. Due to the involvement of multiple objective functions, the fuzzy goal programming method is used to address the multi-objectiveness. Aliahmadi,et al [42] proposed a mathematical model for a multi-echelon closed-loop supply chain network, taking into account pricing decisions and queuing systems in the face of uncertainty. The study focused on the impact of actual demand on pricing decisions for both final and returned products within the supply chain network. The objective of the mathematical model was to maximize the net present value considering uncertain parameters related to potential demand and transportation costs.

3. Mathematical Model

3.1. Network Structure

The structure CLSC (Fig. 1) consists of production centers, retail centers, probabilistic customers, collection centers, and disposing centers. All modeling and written equations in this article are based on this figure.

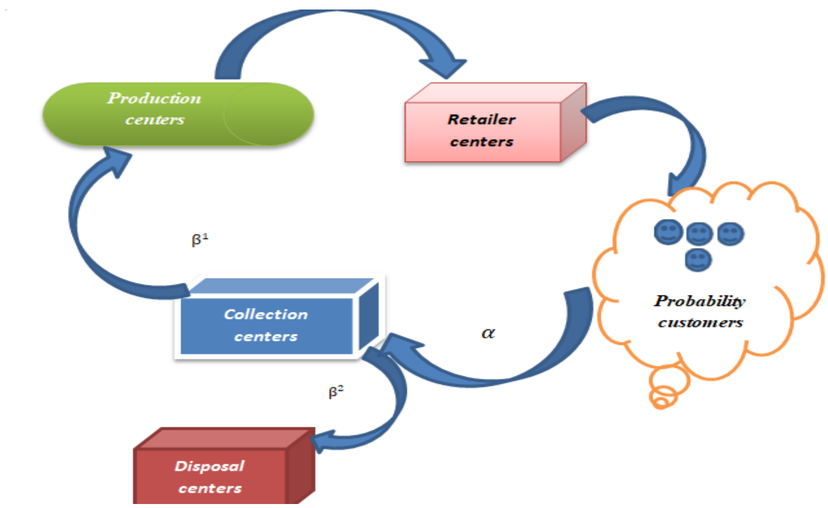

Figure. 1 Schematic Diagram of The Modeled CLSC

In Fig. 1, the customers are chosen probabilistically with lower and upper time bounds and have minimum and maximum coverage radii to reach retailers. Also, retailers have lower and upper bounds such that they have minimum and maximum coverage ranges to provide customer service.

3.2. Model Assumption

- The single-period model is considered;
- Insufficiency is allowed;
- Transportation costs are fixed over a period;
- All customers must receive their services;
- Every customer can visit more than one retail center to receive services;
- The customers' and retailers' motion type are either Rectangular, Euclidean, Euclidean Square, or Chebyshev;
- The motion happens on a page;
- Based on the relocation distance, the transportation time is constant and invariable;
- All chain parameters and variables are definite, excluding the customer place, customer coverage time, retailer coverage distance, and customer's demand;
- No cost is considered for keeping the goods;

- The retail coverage distance for random customers is not constant at each stage;
- The customer coverage time of retailers is not constant at every stage;
- The quantity of goods in retailers is always constant.

3.3. Sets

In this section, all the indexes used in modeling the problem are presented.

$p = 1,2, \dots P$	Index of collection centers that have the potential to produce
$j = 1,2, \dots J$	Index of collection of retail centers that have the selling potential
$i = 1,2, \dots I$	Index of collection of probabilistic customers
$cc = 1,2, \dots CC$	Index of collection centers that have the potential to collect
$dis = 1,2, \dots DIS$	Index of collection of disposal centers that have the elimination potential
$r = 1,2, \dots R$	Index of the produced goods

e, e'= The set of all echelons $(e, e' \in \{p\ , j, i, cc, dis\})$

k, k'= The set of facilities in the echelon $(k_e, k'_e \in \{1, \dots K_e\})$

3.4. Model Parameters

In this section, all the parameters associated with the problem are introduced.

Table 1. Parameters Used in Model

cap_e	Capacity of facility $e \in \{p, j, i, cc, dis\}$
cap_{k_e}	Capacity of facility $k_e \| e \in \{p\ , j, cc, dis\}$
τ_{k_e}	The amount of facility $k_e \| e \in \{p\ , j, cc, dis\}$
v_r	The amount of the type r production
Fc_{k_e}	Fixed cost of the facility $k_e \| e \in \{p\ \}$
FC_j	Fixed cost of the *jth* potential retailer center
FC'_{k_e}	Fixed cost of the facility $k_e \| e \in\in \{cc, dis\}$
L_j	Standard radius of the service distance for the *jth* retailer
T_i	Standard radius of time of service receiving for the *ith* probabilistic costumer
$l_{j,i,r}$	Distance covered by the *jth* retailer transfer to provide service for the *ith* probabilistic customer due to send the type r product

$t_{i,j,r}$	Time covered by the *ith* probabilistic customer spend to gain service for the *jth* retailer due to receive the type r product
e_j	Upper and lower limit values from the standard distance radius
θ_i	Upper and lower limit values from the standard time radius
μ_{i1}	The average horizontal coordinates of the *ith* probabilistic customer
μ_{i2}	The average vertical coordinates of the *ith* probabilistic customer
σ_{i1}^2	The variance of horizontal coordinates of the *ith* probabilistic customer
σ_{i2}^2	The variance of vertical coordinates of the *ith* probabilistic costumer
d_{j1}	Spatial horizontal coordinates of the *jth* retailer
d_{j2}	Spatial vertical coordinates of the *jth* retailer
$\pi_{j,i,r}$	The shortage costs of the type *rth* product from the *jth* potential retailer center to the *ith* probabilistic customer
$\pi'_{i,j,r}$	The shortage costs of the type *rth* product from the *ith* probabilistic customer to the *jth* potential retailer center
α	The percentage of customers returning goods. ($\alpha \leq 1$)
β^1, β^2	The percentage of the products that can be revived in the collection center and eliminated in the disposal center ($\beta^1 + \beta^2 = \alpha$)
$Cost_{k_e k'_{e'},r}$	The relocation cost of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{p , j\})$
$Cost_{j,i,r}$	The relocation cost of the type *r* product from the *jth* potential retailer center to the *ith* probabilistic customer
$Cost'_{i,j,r}$	The relocation cost of the type *r* product from the *ith* probabilistic customer to the *jth* potential retailer center
$Cost'_{k_e k'_{e'},r}$	The relocation cost of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{i, cc, dis, p\})$
$dic_{k_e k'_{e'},r}$	The transferring distance of the type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{p , j\})$

$dic_{j,i,r}$	The distance for transferring type *r* product from the *j* th potential retailer center to the *i* th probabilistic customer.
$dic'_{i,j,r}$	The distance for transferring type r product from the *i* th potential retailer center to the *j* th probabilistic customer.
$dic'_{k_e k'_{e'},r}$	The distance for transferring type *r* product from the facility center k_e to facility center $k_{e'}$ $(e, e' \in \{i, cc, dis, p\})$
$x_{k_e k'_{e'},r}$	The amount of the type *r* product sends from the facility center k_e to facility center $k'_{e'}$ $(e, e' \in \{p\ , j\})$
$x_{j,i,r}$	The amount of probabilistic demand of the type *r* product send from *jth* potential retailer center to the *ith* probabilistic customer
$x'_{i,j,r}$	The amount of probabilistic demand of the type *r* product received the *ith* probabilistic customer from, the *jth* potential retailer center
$x'_{k_e k'_{e'},r}$	The amount of the type *r* product sends from the facility center k_e to facility center $k'_{e'}$ $(e, e' \in \{i, cc, dis, p\})$
$dicR$ $dicED$ $dicEDS$ $dicCH$	Rectangular motion Euclidean motion Euclidean Square motion Chebyshev motion
COV^1 COV^2	Expected distance coverage Expected time coverage

3.5. Decision Variable

In this paper, only one decision variable (0 and 1) is to designate customers to retailers or vice versa.\

$$\boldsymbol{Q_1} = \begin{cases} \mathbf{1} & \boldsymbol{if\ the\ retailer\ is\ allocated\ to\ the\ costumer} \\ \mathbf{0} & \boldsymbol{otherwise} \end{cases}$$

3.6. Model formulation

The mathematical model of this chain consists of two stages. The first stage formulates the mathematical model among probabilistic customers and retailers, while the second stage formulates the entire problem.

Step 1: Calculations are highly effective for retailers and potential customers. Thus, it is presumed that the movement type from retailers to customers or vice versa, and the calculation of retailers' distance coverage radius and customers' time coverage radius are compared using heuristic algorithms. Subsequently, the minimum value is selected and deemed as the output of this section.

$$f = min\left(\left\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min\left(f_{1(j,i)}, f_{3(j.i)}\right)\right\} \times Q_1 + \left\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,jr} \times Cost'_{i,j,r} \times min\left(f_{2(i,j)}, f_{4(i,j)}\right)\right\} \times (1 - Q_1)\right) \quad (1)$$

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$$\theta_i < T_i \qquad \forall i \in I \quad (2)$$

$$e_j < L_j \qquad \forall j \in J \quad (3)$$

$$Q_1 + (1 - Q_1) = 1 \quad (4)$$

Equation (1) outlines the objective function for the initial step involving retailers and probabilistic customers. This process comprises two distinct parts delineated by "{}”. The first part calculates the minimum coverage of expected distance and motion (Rectangular, Euclidean, Euclidean Square, and Chebyshev) for retailers providing services to customers. Additionally, this section specifies the probabilistic customer demand. The second part involves computing the minimum expected time coverage and motion (Rectangular, Euclidean, Euclidean Square, and Chebyshev) for customers availing services from retailers. Ultimately, the minimum costs are compared, and the lowest value is chosen as the output. This output determines whether the retailers deliver the goods or the customers pick them up. Constraint (2) defines the maximum time coverage radius for the customer, while Constraint (3) specifies the maximum motion radius for retailers. Finally, Constraint (4) pertains to the selection of the decision variable.

Step 2: In this step, according to Fig. 1 of the modeling process, the input and output ports of the desired chain are calculated. The first objective function shows the value of the first target function (displacement cost), and the second objective function shows the profits from the sale of goods. For more information, see the related Equation $f_{1(j,i)}$, $f_{2(i,j)}, f_{3(j.i)}, f_{4(i,j)}$E($S$) and E ($S'$) in APPENDIX.

$$TOTAL\ OBJECT1 = min\left(\left\{\sum_{k_e \in \{p\}} Fc_{k_e} + \sum_{k_e \in \{p,j\}} \sum_r x_{k_e k'_{e'},r} \times Cost_{k_e k'_{e'},r} \times dic_{k_e k'_{e'},r}\right\} + min\{f\} + \left\{\sum_{k_e \in \{cc,dis\}} FC'_{k_e} + \sum_{k_e \in \{i,cc,dis,p\}} \sum_r x'_{k_e k'_{e'},r} \times Cost'_{k_e k'_{e'},r} \times dic'_{k_e k'_{e'},r}\right\}\right) \quad (5)$$

Equation (5) represents the objective function of the process, which consists of 3 parts, each separated by a "{}" from the others. Part 1 includes the sum of the fixed and the variable costs of the transfer of goods, which are shipped from the production centers to the retailer centers. Part 2 is calculated using Equation (1). Finally, Part 3 includes the entire fixed and variable cost of products returned by customers to the collection centers. Afterward, fixed and variable costs of the transfer of goods from collection centers to repair and disposing centers are calculated. Finally, calculated fixed and variable costs of the transfer of goods from the repair center to distribution and warehouse and disposing centers are computed.

$$TOTAL\ OBJECT2 = max(SA_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times max\{(E(x_{j,i,r}) - cap_j), 0\} \times Q_1) + (BU'_{i,j,r} \times min\{E(x'_{i,j,r}), cap_j\} - \pi'_{i,j,r} \times max[(E(x'_{i,j,r}) - cap_j), 0] \times 1 - Q_1) \quad (6)$$

Equation (6) presents the Profit function of the probabilistic customer by uncertain demand.

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$$\sum_{k_e|e\in\{p,j,cc,dis\}} \tau_{k_e,r} \leq cap_{e\in\{p,j,cc,dis\}} \qquad \forall r \quad (7)$$

$$\sum_r \tau_{k_e,r} \leq cap_{k_e} \quad \forall k_e|e \in \{p, j, cc, dis\} \qquad (8)$$

$$\sum_{k_e|e\in\{p,,j,cc,dis\}} \tau_{k_e,r} \leq v_r \qquad \forall r \quad (9)$$

$$\sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,p\})} x'_{k_e k'_{e'},r} + \sum_{k_e,k'_{e'}|e,e'\in\{p,j\}} x_{k_e k'_{e'},r} \leq \sum_{k_e|e\in\{p\}} \tau_{k_e,r} \qquad \forall r \quad (10)$$

$$\sum_r x_{k_e k'_{e'},r} \leq \quad \forall x_{k_e k'_{e'}} \in (e, e' \in \{p, j\}), \forall \tau_{k_e} | e \in \{j\} \qquad (11)$$

$$\sum_{x_{k_e k'_{e'}} \in(e,e'\in\{p,j\}),r} x_{k_e k'_{e'},r} = \left(\sum_{i=1}^{I}\sum_{j=1}^{J} E(x_{j,i,r})\right) \times Q_1 + \left(\sum_{j=1}^{J}\sum_{i=1}^{I} E(x'_{i,j,r})\right) \times (1 - Q_1) \qquad \forall r \quad (12)$$

$$\left(\sum_i^I\sum_j^J \alpha_{j,i,r} \times \sum_{i=1}^{I}\sum_{j=1}^{J} E(x_{j,i,r})\right) \times Q_1 + \left(\sum_j^J\sum_i^I \alpha_{i,j,r} \times \sum_{j=1}^{J}\sum_{i=1}^{I} E(x'_{i,j,r})\right) \times (1 - Q_1) = \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} \qquad \forall r \quad (13)$$

$$\sum_i^I\sum_j^J \alpha_{j,i,r} \times Q_1 + \sum_j^J\sum_i^I \alpha_{i,j,r} \times (1 - Q_1) \leq 1 \qquad \forall r \quad (14)$$

$$\sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} \leq \sum_{k_e|e\in\{cc\}} \tau_{k_e,r} \qquad \forall r \quad (15)$$

$$\sum_{cc=1}^{CC}\sum_{i=1}^{I} \beta^1_{i,cc,r} \times \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} = \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,p\})} x'_{k_e k'_{e'},r} \qquad \forall r \quad (16)$$

$$\sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,p\})} x'_{k_e k'_{e'},r} \leq \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} \qquad \forall r \quad (17)$$

$$\sum_{cc}^{CC}\sum_i^I \beta^2_{i,cc,r} \times \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} = \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,dis\})} x'_{k_e k'_{e'},r} \qquad \forall r \quad (18)$$

$$\sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,dis\})} x'_{k_e k'_{e'},r} \leq \sum_{k_e|e\in\{dis\}} \tau_{k_e,r} \qquad \forall r \quad (19)$$

$$\sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,dis\})} x'_{k_e k'_{e'},r} \leq \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} \qquad \forall r \quad (20)$$

$$\sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,p\})} x'_{k_e k'_{e'},r} + \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{cc,dis\})} x'_{k_e k'_{e'},r} = \sum_{x'_{k_e k'_{e'},r}\in(e,e'\in\{i,cc\})} x'_{k_e k'_{e'},r} \qquad \forall r \quad (21)$$

Constraint (7) represents the maximum capacity of all facility centers. Constraint (8) indicates the maximum capacity. Constraint (9) declares the maximum production amount, r, in each facility center. Constraint (10) shows the maximum capacity of goods entering the production centers.

Constraint (11) indicates the maximum entry capacity of each retailer center. Constraint (12) ensures the balance of goods entering and leaving retail centers. Based on Algorithms 1 and 2, this constraint determines whether the retailers send goods to the customers or the customers go to the retailers to receive the goods. Constraint (13), depending on the decision variable, shows the percentage of goods returned by customers. Constraint (14) sets the total percentage of goods returned by customers to a maximum of 1. Constraint (15) represents the maximum capacity of goods entering the collection centers from all customers. Constraint (16) shows the percentage of goods shipped from the collection center to the production centers. Constraint (17) states that the maximum number of goods in the collection center equals the number of returned goods. Constraint (18) gives the percentage of goods sent from the total collection center to the disposal centers. Constraint (19) shows that the maximum number of goods sent from the collection centers to the disposal centers is equal to the maximum capacity of the disposal centers. Constraint (20) asserts that the maximum number of destroyed goods is equal to the number of returned goods. Finally, Constraint (21) presents the balance of goods entry.

4. Solution Approach

Additionally, the proposed model not only reduces costs for retailers by selecting the best location but also decreases the carbon dioxide emissions for both retailers and customers. In this model, retailers can be chosen to provide services, while customers can also be selected to receive services. The expected distances between customers and retailers are calculated based on different movement methods (Rectangular, Euclidean, Euclidean Square, and Chebyshev) due to the probabilistic nature of customers. These values are then compared with the MECD of retailers, as displayed in Algorithm 1, and the minimum value is selected. Similarly, the minimum value is used to assign customers to retailers based on their movement methods and compared with MECT, presented in Algorithm 2. Ultimately, the allocation and service provision method is determined by choosing the minimum cost from the two methods mentioned above.

Algorithm 1: Assigning retailers to customers

***Step 1**: Initialization*

-Generate the average longitudinal and transverse to the number of probabilistic customers

$\boldsymbol{\mu}_{11}, \boldsymbol{\mu}_{21}, \dots, \boldsymbol{\mu}_{I1}.$

$\boldsymbol{\mu}_{12}, \boldsymbol{\mu}_{22}, \dots, \boldsymbol{\mu}_{I2}.$

***Step 2**: Computing expected distance and cost*

- Compute expected distance based on the type of sending the goods from retailers to probabilistic customers (Rectangular, Euclidean, Euclidean Square, and Chebyshev)

-Calculate the cost of sending goods from retailers to potential customers using the second step

Step 3**: **Computing distance coverage radius

- Computes $the\ maximum\ distance$ *based on the second step*

$$maximum\ distance = max\left(E\left[\left(dicR_{j,i}\right),\left(dicED_{j,i}\right),\left(dicEDS_{j,i}\right),\left(dicCH_{j,i}\right)\right]\right)$$

$l_{j,i,r}$=$rand([1\ ,maximum\ distance])_{J\times I\times R}$

Step 4**: **Computing maximum and the minimum distance coverage radii

-For all retailers to provide services, calculate the minimum and the maximum distance coverage radius separately;

Min and Max = [$L_j - e_j \quad L_j + e_j$ *]*

Step 5: assign retailer

If:

$\left(l_{j,i,r}\right) \leq \left[L_j - e_j\right]$ *Then, calculate* $\left\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)\right\}$ *and assign the retailer to the* $max\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)$

$L_j - e_j < (l_{j,i,r}) \leq L_j$, *then, calculate* $\left\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)\right\}$ *and assign the retailer to the* $max\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)$

$L_j < (l_{j,i,r}) \leq L_j + e_j$, *then, calculate* $\left\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)\right\}$ *and assign the retailer to the* $max\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)$

$\left(l_{j,i,r}\right) > L_j + e_j$, *then calculate* $\left\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times Cost_{j,i,r} \times min\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)\right\}$ *and assign the retailer S to the* $max\left(E\left[dic_{j,i,r}\right], E\left[COV^1\left(l_{j,i,r}\right)\right]\right)$

Algorithm 2*: Assigning customers to retailers*

Step 1**: **Initialization

Generate the average longitudinal and transverse to the number of probabilistic customers

$\boldsymbol{\mu}_{11}, \boldsymbol{\mu}_{21}, \dots, \boldsymbol{\mu}_{I1}$.

$\boldsymbol{\mu}_{12}, \boldsymbol{\mu}_{22}, \dots, \boldsymbol{\mu}_{I2}$.

Step 2*: *Computing expected distance and cost

Compute expected distance based on the type of sending the goods from retailers to probabilistic customers (Rectangular, Euclidean, Euclidean Square, and Chebyshev),

Calculate the cost of sending goods from retailers to potential customers using the second step

Step 3*: *Computing time coverage radius

Using the second step, compute $the\ maximum\ distance$.

$$MaxTime = max\left(E\left[\left(dicR_{i,j}\right),\left(dicED_{i,j}\right),\left(dicEDS_{i,j}\right),\left(dicCH_{i,j}\right)\right]\right)$$

$t_{i,j,r}$=$rand([1\ ,\ MaxTime])_{I\times J\times R}$

Step 4*: *Computing maximum and the minimum time coverage radius

For all retailer service providers, calculate the minimum and the maximum time coverage radius separately;

Min and max= $[T_i - \theta_i \quad T_i + \theta_i]$

Step 5: assign customer

if:

$\left(t_{i,j,r}\right) \leq [T_i - \theta_i]$ *then calculate* $\left\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)\right\}$ *And assign the customer to the max* $\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)$

$T_i - \theta_i < \left(t_{i,j,r}\right) \leq T_i$ *then calculate* $\left\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)\right\}$ *And assign the customer to the max*$\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)$

$T_i < \left(t_{i,j,r}\right) \leq T_i + \theta_i$, *then calculate* $\left\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)\right\}$ *And assign the customer to the max* $\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)$

$\left(t_{i,j,r}\right) > T_i + \theta_i$ *then calculate* $\left\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times min\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)\right\}$ *And assign the customer to the max* $\left(\left[dic_{i,j,r}\right], E\left[COV^2\left(t_{i,j,r}\right)\right]\right)$

The minimum cost is chosen by comparing the outputs of Algorithm 1 and Algorithm 2.

5. Non-Sorting-Genetic Algorithm II

The application of meta-heuristic algorithms can yield significant benefits in addressing complex problems. As a result, the utilization of NSGA-II for solving unconstrained multi-objective problems is rapidly gaining traction. Pasandideh,et al [43]. this study, the algorithm is employed to address the general model of the SC, given its dual-objective nature and numerous constraints. Figure 2 illustrates the flowchart of this algorithm. Garg,et al [44].

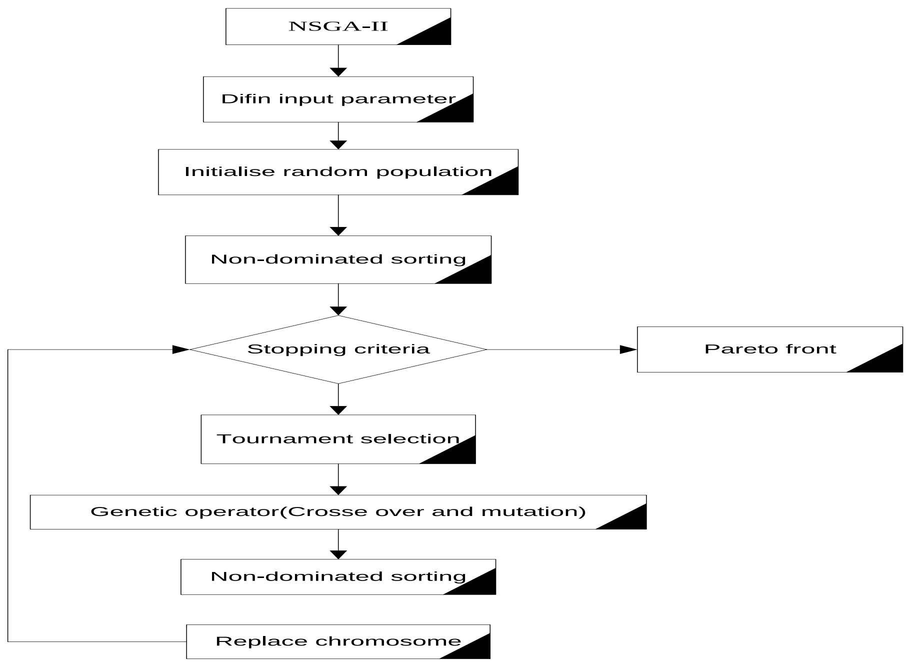

Figure 2. Flow Chart of the NSGA-II

In addition, a Taguchi method is used to set the parameters of these algorithms to enhance their performance. Table 2 shows proposed values for NSGA II algorithm parameters. see the related data in appendix.

5.1. Proposed chromosome

The structure of the chromosome is defined to be consisting of several components, in which the variable related to location and survivors are considered as one part. The variables in this part are defined as is shows figure 3.

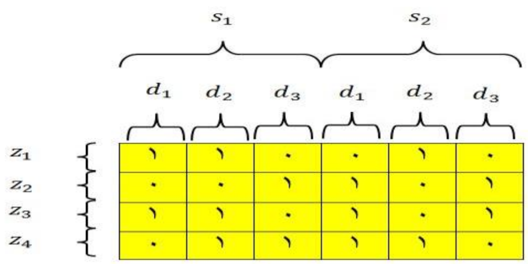

Figure 3. Proposed Chromosome

Figure 4 shows the intersection of the proposed chromosome. As can be seen, a single point intersection has been used. In this form, a point is randomly selected and the corresponding genes are moved

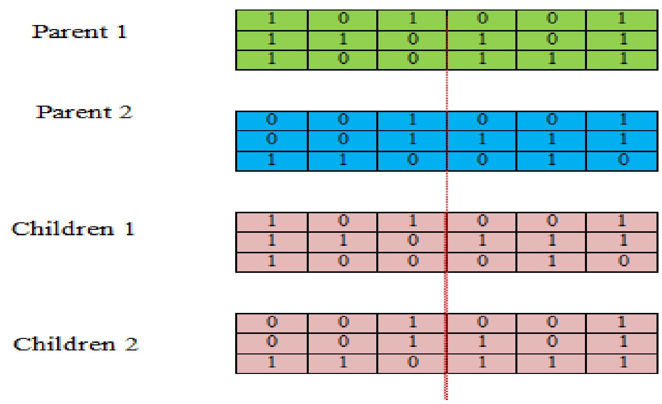

Figure 4. Proposed Chromosome

5.2. Mutation Operation

Figure 5 shows the mutation operator. For this purpose, a row is selected as desired and the selected row is reversed.

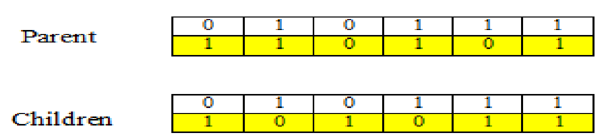

Figure 5. Mutation Operation

To estimate the algorithm parameters, the result of 100 experiments designed for this problem shows that in each experiment, the algorithm parameters change and the results change accordingly. Figure 6 shows the results of the Taguchi approach for estimating parameters.

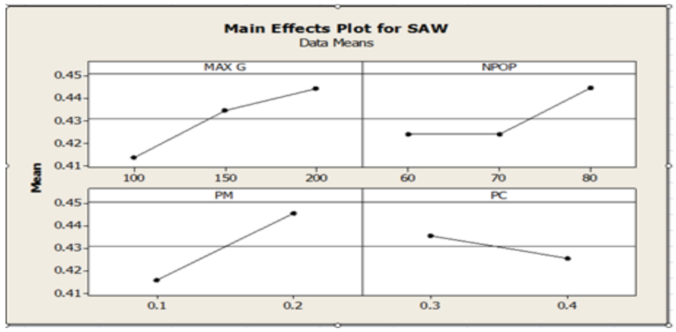

Figure 6. Taguchi Parameter Adjustment

In addition, a Taguchi method is used to set the parameters of these algorithms to enhance their performance. Table 2 shows proposed values for NSGA II algorithm parameter

Table 2. Proposed Values for NSGA II Algorithm Parameters

mutation	Cross over	Pop number	Max number
0.2	0.4	80	200

6. Numerical example

Fig. 1 presents a numerical example for the CLSC model to understand the problem model. This problem is solved using MATLAB R2018b coding.

7. Computational results

The CLSC, as a multi-objective matter, stands as a crucial facet of SC problems. A key objective in these scenarios involves reducing retailer service distance or cutting customer travel time to reach service centers. In this study, a specific instance of CLSC problems, the focus lies on presenting heuristic allocation algorithms. It emphasizes retailers with known coordinates and their coverage distance for service provision. Additionally, customers possess probabilistic coordinates and visitation time for retail centers. This model achieves optimal allocation by simultaneously evaluating retailer distances and expected coverage, alongside the time and expected coverage of probabilistic customers. Also, the best places of retailers are discovered using the NSGA-II algorithm. Afterward, the distance coverage radius between retailers and the time coverage radius of the customers is calculated considering the amount of standard radius, upper and lower bounds of each of the retailers and customers. To prevent further dissemination in solving this example, we held the potential location search range for probabilistic customers within [4000, 7000] and the optimal search location for retail centers within [3000, 7000] spans. Thus, the optimal coordinates of retailers are calculated in the same span.

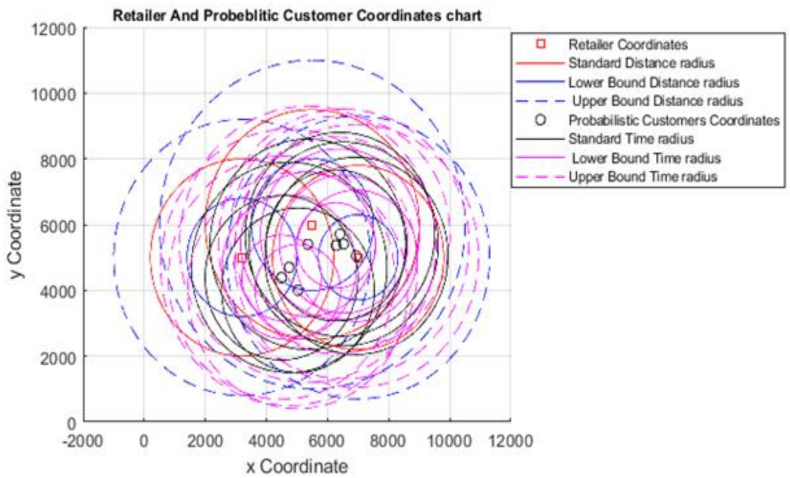

Figure 7. Initial Coordinates OF Retailers and Probabilistic Customers and Standard Lower and Upper Coverage Radius

Initial Coordinates of Retailers and Probabilistic Customers and Standard Lower and Upper Coverage Radius are essential for understanding the distribution of potential customers and the reach of each retailer. Analyzing these coordinates allows us to establish the standard lower and upper coverage radius for each retailer, enabling them to optimize their efforts. This data will offer valuable insights into customer behavior and preferences, empowering retailers to better meet the needs of their target audience.

Fig. 8 Considering that customer location coordinates are random and include any value, the red star points in this figure depict the probabilistic customer coordinates. The optimal solution point from the heuristic algorithm is also shown with a black star. In the third part, this point is compared with random points and guided to the best point according to the algorithm, represented by a square containing the red star.

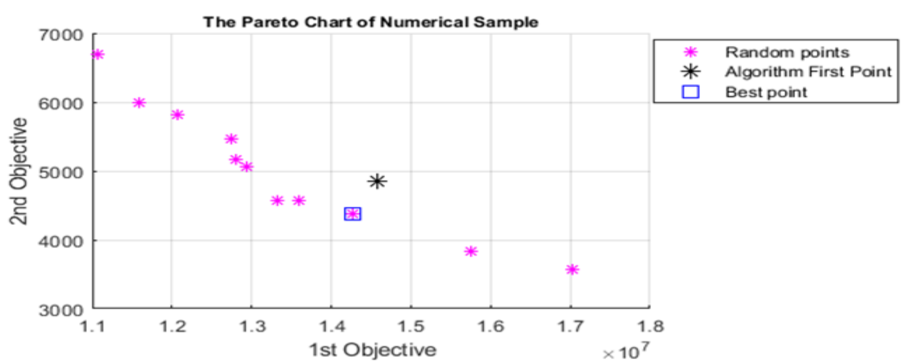

Figure 8. The Pareto Front Chart of Numerical Sample

The results of problem-solving before and after solving with the NSGA-II algorithm are presented in Table 3. The results show that customers should turn to retailers for services. The example results show that both the original and improved algorithms require customers to contact retailers for service.

A comparison table depicts the cost and profit before and after the genetic algorithm's final solution. The algorithm's efficiency and its proximity to solving the problem with the genetic algorithm make it an acceptable solution. The comparison table also indicates that the improved algorithm led to a significant cost reduction and profit increase, demonstrating the genetic algorithm's effectiveness in optimizing the solution. The algorithm's final solution's proximity to the optimal one indicates its efficiency and reliability. The results also emphasize the potential for further algorithm improvements and the need for ongoing monitoring and adjustments to ensure continued optimization.

Table 3. Total Cost and Profit Function

	Results check	Cost and profit calculated by allocating the proposed algorithm before NSGA-II		Cost and profit calculated by allocating the proposed algorithm after NSGA-II	
		Cost function	Profit function	Cost function	Profit function
1	The total cost of the system, if retailers provide services to probabilistic customers	14806589	5082	14385401	4449
2	The total cost of the system, if probabilistic customers refer to retailers for service	14572321	4913	14260093	4311
Conclusion		Customers refer to retailers for services		Customers refer to retailers for services	

Fig. 9 shows the starting coordinates of retailers and potential customers. Additionally, it displays the optimal coordinates calculated from Table 3 that retailers can provide to these customers. The original retailer locations are denoted by red squares, while the random probablistic customer locations are represented by black circles. After the improvement, this algorithm suggests new coordinates for retailers, shown as green squares. Coordinates for retailers, shown as green squares.

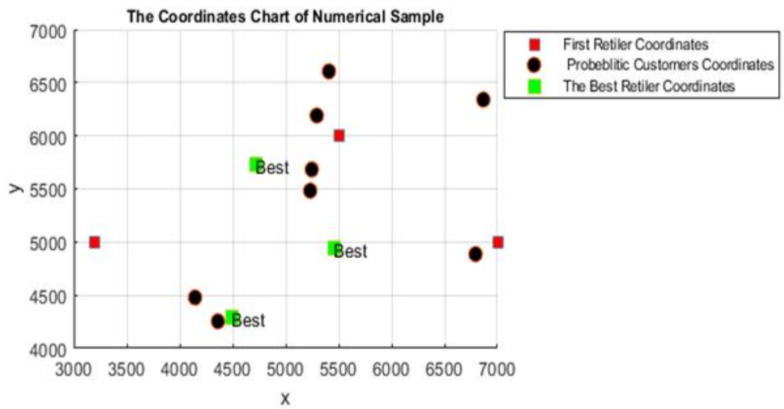

Figure 9. Optimal Coordinated of Retailers

Table 4 presents the coordinates of retailers before and after being solved by NSGA-II. As the range of coordinates for retailers is limited, the optimal solutions have also been chosen within this range.

Table 4. Retailers Coordinates Before and After Solving By NSGA-II

Number Retailer	Coordinates of the retailers before solving with NSGA-II	Coordinates of the retailers after solving with NSGA-II
1	[3200, 5000]	[4489.6, 4288.2]
2	[5500, 6000]	[5457.6, 4939.4]
3	[7000, 5000]	[4715.9, 3572.7]

The results from Table 5 show that after using NSGA-II, the allocation of retailers to customers has improved, ensuring that customers are directed to the most suitable retailers for their needs. This optimization has led to improved efficiency and customer satisfaction, benefiting both retailers and customers.

Table 5. Probabilistic Customers Assigned to Retailers

		Before solving by Nsga2			After solving by Nsga2		
		j=1	j=2	j=3	j=1	j=2	j=3
i=1	r=1	✓				✓	
	r=2		✓			✓	
i=2	r=1			✓			✓
	r=2		✓				✓
i=3	r=1		✓			✓	
	r=2	✓				✓	
i=4	r=1		✓				✓
	r=2	✓					✓
i=5	r=1		✓			✓	
	r=2	✓				✓	
i=6	r=1	✓					✓
	r=2			✓			✓
i=7	r=1			✓		✓	
	r=2			✓		✓	
i=8	r=1		✓		✓		
	r=2	✓			✓		

Table 6 shows the customers' preference for motion or coverage usage before and after the NSGA-II algorithm solution. Prior to implementing the NSGA-II algorithm solution, all customers used coverage options. However, following the NSGA-II algorithm solution, customers incorporated movement in addition to coverage usage, indicating a notable enhancement in customer engagement with these features.

Table 6. Deciding On the Type of Movement or Coverage

		Before solving by Nsga2			After solving by Nsga2		
		j=1	j=2	j=3	j=1	j=2	j=3
i=1	*r=1*	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**
i=2	*r=1*	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
i=3	*r=1*	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
i=4	*r=1*	COV^2	COV^2	COV^2	COV^2	COV^2	**dicCH**
	r=2	COV^2	COV^2	COV^2	COV^2	COV^2	**dicCH**
i=5	*r=1*	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	dicED	COV^2	**COV^2**
i=6	*r=1*	COV^2	COV^2	COV^2	COV^2	dicCH	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	dicCH	**COV^2**
i=7	*r=1*	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	COV^2	**COV^2**
i=8	*r=1*	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**
	r=2	COV^2	COV^2	COV^2	COV^2	dicED	**COV^2**

8. Conclusion

In this paper, a closed-loop supply chain (CLSC) is modeled to obtain the best location of retailers and allocate them to other utilities. The structure of CLSC includes production centers, retailers' centers, probabilistic customers, collection, and disposal centers. In the first step, considering the probabilistic place of customers and the fixed location of retailers, the expected distance is calculated at the first step according to the type of movement (Rectangular, Euclidean, Ecclesiastical Square, And Chebyshev). Next, based on the coverage radius, the distance between retailers and the time of probabilistic customers is calculated probabilistically by integral calculations. Additionally, customers were allocated to retailers or vice versa by presenting Algorithms 1 and 2. In the second step, which is the general solution to the problem, the NSGA-II algorithm is applied. The results of applying the model to the studied example indicate that concerning costs and profit. Based on these results, customers are recommended to refer to retailers to receive services. Moreover, the type of motion was hinted at considering the calculated expected coverage time. Furthermore, new coordinates are calculated for retailers such that to provide the lowest cost for customers and enable the optimal allocation of retailers to customers. Finally, routing, relocating time, probabilistic inventory in warehouse can be possible themes for future research using various scenarios in a time window.

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APPENDIX:

$f_{1(j,i)} = \min\left(E\left[dic_{j,i}\right]\right) = min\left(E\left[\left(dicR_{j,i}\right),\left(dicED_{j,i}\right),\left(dicEDS_{j,i}\right),\left(dicCH_{j,i}\right)\right]\right)$

$$= min\left[\left(\sum_{\boldsymbol{j=1}}^{\boldsymbol{J}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{I}}\left(\left|d_{j1}-\mu_{i1}\right|+\left|d_{j2}-\mu_{i2}\right|\right)\right),\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left(\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}}\right)\right)\right),\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left(d_{j1}-\mu_{i1}\right)^2+\sigma_{i1}^2+\left(d_{j2}-\mu_{i2}\right)^2+\sigma_{i2}^2\right)\right),\left(max\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left|d_{j1}-\mu_{i1}\right|,\left|d_{j2}-\mu_{i2}\right|\right)\right)\right]$$

$f_{2(i,j)} = \min\left(E\left[dic_{i,j}\right]\right) = min\left(E\left[\left(dicR_{i,j}\right),\left(dicED_{i,j}\right),\left(dicEDS_{i,j}\right),\left(dicCH_{i,j}\right)\right]\right)$

$$= min\left[\left(\sum_{\boldsymbol{i=1}}^{\boldsymbol{J}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{I}}\left(\left|\mu_{i1}-d_{j1}\right|+\left|\mu_{i2}-d_{j2}\right|\right)\right),\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left(\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}}\right)\right)\right),\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left(\mu_{i1}-d_{j1}\right)^2+\sigma_{i1}^2+\left(\mu_{i2}-d_{j2}\right)^2+\sigma_{i2}^2\right)\right),\left(max\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left|\mu_{i1}-d_{j1}\right|,\left|\mu_{i2}-d_{j2}\right|\right)\right)\right]$$

Lemma1: The following equations are always confirmed.

$$\left[dicR_{j,i}\right] = \sum_{\boldsymbol{j=1}}^{\boldsymbol{J}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{I}}\left(\left|d_{j1}-\mu_{i1}\right|+\left|d_{j2}-\mu_{i2}\right|\right)$$

$$\left[dicR_{i,j}\right] = \sum_{\boldsymbol{i=1}}^{\boldsymbol{J}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{I}}\left(\left|\mu_{i1}-d_{j1}\right|+\left|\mu_{i2}-d_{j2}\right|\right)$$

Proof: a_{i1} *and* a_{i2} are independent of each other. As a result, we have the independence of customers of this model.

$$E\left[dic_{R_{j,i}}\right] = E\left[l\left(d_j,a_i\right)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} l\left(d_j,a_i\right)f_i\left(a_i\right)la_{i1}la_{i2}$$
$$\int_{-\infty}^{+\infty}\left(\left|d_{j1}-a_{i1}\right|f_i\left(a_i\right)la_{i1}la_{i2}+\int_{-\infty}^{+\infty}\left|d_{j2}-a_{i2}\right|f_i\left(a_i\right)la_{i1}la_{i2}\right) =$$
$$\int_{-\infty}^{+\infty}\left(\left(\left|d_{j1}-a_{i1}\right|f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right)+\int_{-\infty}^{+\infty}\left(\left(\left|d_{j2}-a_{i2}\right|f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right) =$$
$$\left[\left|\left(\left[\int_{-\infty}^{+\infty}\left(d_{j1}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]-\left[\left(a_{j1}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]\right)\right|\right]+\left[\left|\left(\left[\int_{-\infty}^{+\infty}\left(d_{j2}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]-\left[\left(a_{j2}f_i\left(a_i\right)la_{i1}\right)f_i\left(a_i\right)la_{i2}\right]\right)\right|\right]$$
$$=\left[\left|d_{j1}-\mu_{i1}\right|+\left|d_{j2}-\mu_{i2}\right|\right]$$

Lemma 2: The following equations are always confirmed **Altınel,et al [45].**

$$\boldsymbol{E\left[dicED_{j,i}\right]} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{J}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{I}}\left[\left(\sqrt{\left(\boldsymbol{d_{j1}}-\boldsymbol{\mu_{i1}}\right)^{\boldsymbol{2}}+\left(\boldsymbol{d_{j2}}-\boldsymbol{\mu_{i2}}\right)^{\boldsymbol{2}}}\right)+\frac{\boldsymbol{1}}{\boldsymbol{2}}\left(\frac{\boldsymbol{\sigma_{i1}^2+\sigma_{i2}^2}}{\sqrt{\left(\boldsymbol{d_{j1}-\mu_{i1}}\right)^{\boldsymbol{2}}+\left(\boldsymbol{d_{j2}-\mu_{i2}}\right)^{\boldsymbol{2}}}}\right)\right]$$

$$\boldsymbol{E\left[dicED_{i,j}\right]} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{J}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{I}}\left[\left(\sqrt{\left(\boldsymbol{\mu_{i1}}-\boldsymbol{d_{j1}}\right)^{\boldsymbol{2}}+\left(\boldsymbol{\mu_{i2}}-\boldsymbol{d_{j2}}\right)^{\boldsymbol{2}}}\right)+\frac{\boldsymbol{1}}{\boldsymbol{2}}\left(\frac{\boldsymbol{\sigma_{i1}^2+\sigma_{i2}^2}}{\sqrt{\left(\boldsymbol{\mu_{i1}-d_{j1}}\right)^{\boldsymbol{2}}+\left(\boldsymbol{\mu_{i2}-d_{j2}}\right)^{\boldsymbol{2}}}}\right)\right]$$

Lemma 3: The following equations are always confirmed**.**

$E\left[dicEDS_{j,i}\right]=\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\left(d_{j1}-\mu_{i1}\right)^2+\sigma_{i1}^2+\left(d_{j2}-\mu_{i2}\right)^2+\sigma_{i2}^2\right)$

$E\left[dicEDS_{i,j}\right]=\sum_{i=1}^{J}\sum_{j=1}^{I}\left(\left(\mu_{i1}-d_{j1}\right)^2+\sigma_{i1}^2+\left(\mu_{i2}-d_{j2}\right)^2+\sigma_{i2}^2\right)$

Proof:

$E\left[l\left(d_j,a_i\right)\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}l\left(d_j,a_i\right)f_i(a_i)la_{i1}la_{i2}$

The Parameters a_{i1},a_{i2} are independent of each other. As a result, the customers of this model are independent.

$\boldsymbol{f_j=(a_{i1}\times a_{i2})=f_{i1}(a_{i1})\times f_{i2}(a_{i2})}$

$E\left[l\left(d_j,a_i\right)\right]=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left(d_{j1}-a_{i1}\right)^2+\left(d_{j2}-a_{i2}\right)^2\right]f_i(a_{i1})\times f_i(a_{i2})la_{i1}la_{i2}$
$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left(d_{j1}-a_{i1}\right)^2 f_i(a_{i1})\times f_i(a_{i2})la_{i1}la_{i2}+\left(d_{j2}-a_{i2}\right)^2 f_i(a_{i1})\times f_i(a_{i2})la_{i1}la_{i2}\right]=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left({d_{j1}}^2+{a_{i1}}^2-2d_{j1}a_{i1}\right)\times f_i(a_{i1})\times f_i(a_{i2})la_{i1}la_{i2}\quad+\left({d_{j2}}^2+{a_{i2}}^2-\ 2d_{j2}a_{i2}\right)\times f_i\left(\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}d_{j1}^2\times f_i(a_{i1})la_{i1}\times f_i(a_{i2})la_{i2}+\int_{-\infty}^{+\infty}a_{i1}^2\times f_i(a_{i1})la_{i1}\times f_i(a_{i2})la_{i2}-\int_{-\infty}^{+\infty}\left(2d_{j1}a_{i1}\right)\times f_i(a_{i1})la_{i1}\times f_i(a_{i2})la_{i2}+\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}d_{j2}^2\times f_i(a_{i1})la_{i1}\times f_i(a_{i2})la_{i2}+\int_{-\infty}^{+\infty}a_{i2}^2\times f_i(a_{i1})la_{i1}\times f_i(a_{i2})la_{i2}-\int_{-\infty}^{+\infty}2d_{j2}a_{i2}\times f_i(a_{i1})la_{i1}\times f_i(a_{i2})la_{i2}\right]a_{i1})\times f_i(a_{i2}\right)la_{i1}la_{i2}\right]=$
$\left[d_{j1}^2+\left(\sigma_{i1}^2+\mu_{i1}^2\right)-2d_{j1}\mu_{i1}\right]+\left[d_{j2}^2+\left(\sigma_{i2}^2+\mu_{i2}^2\right)-2d_{j2}\mu_{i2}\right]$

On the other hand, the following statements are always valid:

$$\int_{-\infty}^{+\infty}a_{ik}f_{ik}(a_{ik})la_{ik}=\mu_{ik}$$

$$\int_{-\infty}^{+\infty}{a_{ik}}^2 f_{ik}(a_{ik})la_{ik}=\sigma_{ik}^2+\mu_{ik}^2$$

$$\int_{-\infty}^{+\infty}f_{ik}(a_{ik})la_{ik}=1$$

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Lemma 4: The following equations are always confirmed Drezner,et al [46].

$f_{3(j,i.r)}$=$Max\ E\left(COV^1\left(l_{j.i.r}\right)\right)$ =

$$\begin{cases} 1 & l_{jir} \le L_j - \mathrm{e}_j \\ \left(\frac{1+\left(\frac{L_j-l_{jir}}{\mathrm{e}_j}\right)}{2}\right)+2\left(\frac{L_j-l_{jir}}{\mathrm{e}_j}\right)\times ln2-\frac{1}{2}\left(\left(\left(\frac{L_j-l_{jir}}{\mathrm{e}_j}+1\right)^2\times ln\left(1+\left(\frac{L_j-l_{jir}}{\mathrm{e}_j}\right)\right)\right)-\left(\left(\frac{L_j-l_{jir}}{\mathrm{e}_j}\right)^2\times ln\left(\frac{L_j-l_{jir}}{\mathrm{e}_j}\right)\right)\right) & L_j-\mathrm{e}_j < l_{jir} \le L_j \\ \left(\frac{1-\left(\frac{l_{jir}-L_j}{\mathrm{e}_j}\right)}{2}\right)-2\left(\frac{l_{jir}-L_j}{\mathrm{e}_j}\right)\times ln2+\frac{1}{2}\left(\left(\left(\frac{l_{jir}-L_j}{\mathrm{e}_j}+1\right)^2\times ln\left(1+\left(\frac{l_{jir}-L_j}{\mathrm{e}_j}\right)\right)\right)-\left(\left(\frac{l_{jir}-L_j}{\mathrm{e}_j}\right)^2\times ln\left(\frac{l_{jir}-L_j}{\mathrm{e}_j}\right)\right)\right) & L_j < l_{ijr} \le L_j+\mathrm{e}_j \\ 0 & l_{ijr} > L_j+\mathrm{e}_j \end{cases}$$

Lemma 5: The following equations are always confirmed Drezner, Drezner and Goldstein [46].

$f_{4(i.j.r)}$= $Max\ E\left(COV^2\left(t_{i.j.r}\right)\right)$=

$$\begin{cases} 1 & t_{ijr} \le T_i - \theta_i \\ \left(\frac{1+\left(\frac{T_i-t_{ijr}}{\theta_i}\right)}{2}\right)+2\left(\frac{T_i-t_{ijr}}{\theta_i}\right)\times ln2-\frac{1}{2}\left(\left(\left(\frac{T_i-t_{ijr}}{\theta_i}+1\right)^2\times ln\left(1+\left(\frac{T_i-t_{ijr}}{\theta_i}\right)\right)\right)-\left(\left(\frac{T_i-t_{ijr}}{\theta_i}\right)^2\times ln\left(\frac{T_i-t_{ijr}}{\theta_i}\right)\right)\right) & T_i-\theta_i < t_{ijr} \le T_i \\ \left(\frac{1-\left(\frac{t_{ijr}-T_i}{\theta_i}\right)}{2}\right)-2\left(\frac{t_{ijr}-T_i}{\theta_i}\right)\times ln2+\frac{1}{2}\left(\left(\left(\frac{t_{ijr}-T_i}{\theta_i}+1\right)^2\times ln\left(1+\left(\frac{t_{ijr}-T_i}{\theta_i}\right)\right)\right)-\left(\left(\frac{t_{ijr}-T_i}{\theta_i}\right)^2\times ln\left(\frac{t_{ijr}-T_i}{\theta_i}\right)\right)\right) & T_i < t_{ijr} \le T_i+\theta_i \\ 0 & t_{ijr} > T_i+\theta_i \end{cases}$$

Lemma 6: The following equations are always confirmed.

$$E\left[dicCH_{j,i}\right] = max\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left|d_{j1}-\mu_{i1}\right|,\left|d_{j2}-\mu_{i2}\right|\right)$$

$$E\left[dicCH_{i,j}\right] = max\left(\sum_{i=1}^{J}\sum_{j=1}^{I}\left|\mu_{i1}-d_{j1}\right|,\left|\mu_{i2}-d_{j2}\right|\right)$$

Proof:

$$E\left[l\left(\boldsymbol{d}_j,\boldsymbol{a}_i\right)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(MAX\left(\left|d_{j1}-a_{i1}\right|,\left|d_{j2}-a_{i2}\right|\right)f_i(a_i)la_{i1}la_{i2}\right) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} MAX\ \left(\left(\left|d_{j1}-a_{i1}\right|f_i(a_i)la_{i1},\left|d_{j2}-a_{i2}\right|\right)f_i(a_i)la_{i2}\right) =$$

$$MAX\left(\left|\int_{-\infty}^{+\infty}d_{j1}f_i(a_i)la_{i1}\right|-\left|\int_{-\infty}^{+\infty}a_{i1}f_i(a_i)la_{i1}\right|\right),\left(\left|\int_{-\infty}^{+\infty}d_{j2}f_i(a_i)la_{i2}\right|-\left|\int_{-\infty}^{+\infty}a_{i2}f_i(a_i)la_{i2}\right|\right) =$$
$$MAX\left(\sum_{j=1}^{J}\sum_{i=1}^{I}\left|d_{j1}-\mu_{i1}\right|,\left|d_{j2}-\mu_{i2}\right|\right)$$

Lemma 7: The following equations are always confirmed **Altınel, Durmaz [45].**

$$E\left[dicED_{j,i}\right] = \sum_{j=1}^{J}\sum_{i=1}^{I}\left[\left(\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(d_{j1}-\mu_{i1}\right)^2+\left(d_{j2}-\mu_{i2}\right)^2}}\right)\right]$$

$$E\left[dicED_{i,j}\right] = \sum_{i=1}^{J}\sum_{j=1}^{I}\left[\left(\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}\right)+\frac{1}{2}\left(\frac{\sigma_{i1}^2+\sigma_{i2}^2}{\sqrt{\left(\mu_{i1}-d_{j1}\right)^2+\left(\mu_{i2}-d_{j2}\right)^2}}\right)\right]$$

Lemma 8: The following equations are always confirmed.

$$E(S) = Cost_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times max\{(E(x_{j,i,r}) - cap_j), 0\}$$

$$E(S') = Cost'_{i,j,r} \times min\{E(x'_{i,j,r}), cap_j\} - \pi_{i,j,r} \times max[(E(x'_{i,j,r}) - cap_j), 0]$$

PROOF:

$$E(S) = Cost_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times \max\{(E(x_{j,i,r}) - cap_j), 0\}$$

$$= Cost_{j,i,r} \times \left[\int_0^{cap_j} x_{j,i,r} \times f_x(x_{j,i,r})dx + \int_0^{\infty} cap_j \times f_x(x_{j,i,r})dx\right] - \pi_{j,i,r} \times \left[\int_{cap_j}^{\infty}(x_{j,i,r} - cap_j) \times f(x_{j,i,r})dx\right]$$

$$= Cost_{j,i,r} \times min\{E(x_{j,i,r}), cap_j\} - \pi_{j,i,r} \times max\{(E(x_{j,i,r}) - cap_j), 0\}$$